Name: Richard

Quiz 22

MATH 201

April 22, 2025

1. Find the Taylor polynomial $p_4(x)$ centered at 0 (i.e, the Maclauren polynomial) for $f(x) = e^{2x}$.

$f^{(0)}(x) = e^{2x}$ $\qquad f^{(0)}(0) = e^{2\cdot 0} = 1$

$f^{(1)}(x) = 2e^{2x}$ $\qquad f^{(1)}(0) = 2e^{2\cdot 0} = 2$

$f^{(2)}(x) = 4e^{2x}$ $\qquad f^{(2)}(0) = 4e^{2\cdot 0} = 4$

$f^{(3)}(x) = 8e^{2x}$ $\qquad f^{(3)}(0) = 8e^{2\cdot 0} = 8$

$f^{(4)}(x) = 16e^{2x}$ $\qquad f^{(4)}(0) = 16e^{2\cdot 0} = 16$

$$p_4(x) = f^{(0)}(0) + f^{(1)}(0)x + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$$

$$= 1 + 2x + \frac{4}{2}x^2 + \frac{8}{6}x^3 + \frac{16}{24}x^4$$

$$\boxed{p_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4}$$

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QUIZ 22

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1. Find the Taylor polynomial $p_4(x)$ centered at 0 (i.e, the Maclauren polynomial) for $f(x) = \cos(2x)$.

$$f^{(0)}(x) = \cos(2x) \qquad f^{(0)}(0) = \cos(2\cdot 0) = 1$$

$$f^{(1)}(x) = -\sin(2x)\cdot 2 \qquad f^{(1)}(0) = -\sin(2\cdot 0)\cdot 2 = 0$$

$$f^{(2)}(x) = -\cos(2x)\cdot 4 \qquad f^{(2)}(0) = -\cos(2\cdot 0)\cdot 4 = -4$$

$$f^{(3)}(x) = \sin(2x)\cdot 8 \qquad f^{(3)}(0) = \sin(2\cdot 0)\cdot 8 = 0$$

$$f^{(4)}(x) = \cos(2x)\cdot 16 \qquad f^{(4)}(0) = \cos(2\cdot 0)\cdot 16 = 16$$

$$P_4(x) = f^{(0)}(0) + f^{(1)}(0)x + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$$

$$= 1 + 0\cdot x + \frac{-4}{2}x^2 + \frac{0}{3!}x^3 + \frac{16}{4!}x^4$$

$$\boxed{P_4(x) = 1 - 2x^2 + \frac{2}{3}x^4}$$