

1. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{k}$. Test endpoints (if any).

Show all work.

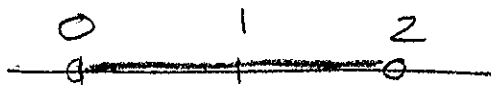
$$\text{Ratio Test: } \lim_{k \rightarrow \infty} \left| \frac{\frac{(-1)^{k+2}(x-1)^{k+1}}{k+1}}{\frac{(-1)^{k+1}(x-1)^k}{k}} \right| = \lim_{k \rightarrow \infty} \left| \frac{k}{k+1} \frac{(x-1)^{k+1}}{(x-1)^k} \right|$$

$$= \lim_{k \rightarrow \infty} \frac{k}{k+1} |x-1| = 1 \cdot |x-1| = |x-1|.$$

For convergence, we must have $|x-1| < 1$

$$-1 < x-1 < 1$$

$$0 < x < 2$$



So the radius of convergence is $R = 1$.

$$\text{Test 0 } \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(0-1)^k}{k} = \sum_{k=1}^{\infty} \frac{(-1)^{2k+1}}{k} = -1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} \dots$$

and this diverges (harmonic)

$$\text{Test 2 } \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2-1)^k}{k} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots$$

and this converges (alternating harmonic)

Thus the interval of convergence is $(0, 2]$

1. Find the interval of convergence of the power series $\sum_{k=1}^{\infty} \frac{(4x-1)^k}{k^2+4}$. Test endpoints (if any). Show work.

$$\text{Ratio Test: } \lim_{k \rightarrow \infty} \left| \frac{\frac{(4x-1)^{k+1}}{(k+1)^2+4}}{\frac{(4x-1)^k}{k^2+4}} \right| = \lim_{k \rightarrow \infty} \left| \frac{(4x-1)^{k+1}}{(4x-1)^k} \frac{k^2+4}{(k+1)^2+4} \right|$$

$$= \lim_{k \rightarrow \infty} |4x-1| \frac{k^2+4}{k^2+2k+5} = |4x-1|$$

For convergence we must have $|4x-1| \leq 1$

$$-1 \leq 4x-1 \leq 1$$

$$0 \leq 4x \leq 2$$

$$0 \leq x \leq \frac{1}{2}$$

$$\text{Test } x=0: \sum_{k=1}^{\infty} \frac{(4 \cdot 0 - 1)^k}{k^2+4} = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2+4} = -\frac{1}{5} + \frac{1}{8} - \frac{1}{13} + \frac{1}{20} - \dots$$

Converges by alternating series test

$$\text{Test } x = \frac{1}{2}: \sum_{k=1}^{\infty} \frac{(4 \cdot \frac{1}{2} - 1)^k}{k^2+4} = \sum_{k=1}^{\infty} \frac{1}{k^2+4} = \frac{1}{5} + \frac{1}{8} + \frac{1}{13} + \frac{1}{20} + \dots$$

Converges by comparison with p -series

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

Thus the interval of convergence is $[0, \frac{1}{2}]$

1. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{k^2 x^{2k}}{k!}$. Test endpoints (if any). Show all work.

Ratio Test $\lim_{k \rightarrow \infty} \left| \frac{\frac{(k+1)^2 x^{2(k+1)}}{(k+1)!}}{\frac{k^2 x^{2k}}{k!}} \right| = \lim_{k \rightarrow \infty} \left| \frac{(k+1)^2 x^{2k+2} k!}{k^2 x^{2k} (k+1)!} \right|$

$$= \lim_{k \rightarrow \infty} \frac{(k+1)^2}{k^2} \cdot \frac{x^{2k+2}}{x^{2k}} \cdot \frac{k!}{(k+1)k!}$$

$$= \lim_{k \rightarrow \infty} \left(\frac{k+1}{k} \right)^2 x^2 \frac{1}{k+1} = 1^2 \cdot x^2 \cdot 0 = 0$$

Because $0 < 1$, we get convergence for any x ,
so the interval of convergence is

$$\boxed{(-\infty, \infty)}$$

1. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{x^{2k+1}}{3^{k-1}}$. Test endpoints (if any). Show all work.

$$\text{Ratio Test } \lim_{k \rightarrow \infty} \left| \frac{\frac{x^{2(k+1)+1}}{3^k}}{\frac{x^{2k+1}}{3^{k-1}}} \right|$$

$$= \lim_{k \rightarrow \infty} \left| \frac{x^{2k+3}}{x^{2k+1}} \cdot \frac{3^{k-1}}{3^k} \right| = \lim_{k \rightarrow \infty} |x^2| \frac{1}{3} = \frac{x^2}{3}$$

For convergence we need $\frac{x^2}{3} < 1$

$$x^2 < 3$$

$$-\sqrt{3} < x < \sqrt{3}$$

$$\text{Test } \sqrt{3}: \sum_{k=1}^{\infty} \frac{\sqrt{3}^{2k+1}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{\sqrt{3}^{2k} \sqrt{3}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{3^k \sqrt{3}}{3^{k-1}}$$

$$= \sum_{k=1}^{\infty} 3\sqrt{3} \leftarrow \text{Diverges}$$

$$\text{Test } -\sqrt{3}: \sum_{k=1}^{\infty} \frac{(-\sqrt{3})^{2k+1}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{-\sqrt{3}^{2k} \sqrt{3}}{3^{k-1}} = \sum_{k=1}^{\infty} \frac{-3^k \sqrt{3}}{3^{k-1}}$$

$$= \sum_{k=1}^{\infty} -3\sqrt{3} \leftarrow \text{Diverges}$$

\therefore Interval of Convergence is $(-\sqrt{3}, \sqrt{3})$