Name: Richard

QUIZ 23

MATH 201
April 24, 2025

1. Find the interval of convergence for the following power series. Test endpoints (if any).

$$\sum_{k=1}^{\infty} \frac{k^{10}(2x-4)^k}{10^k}$$

Ratio Test: $\displaystyle\lim_{k\to\infty}\left|\frac{\frac{(k+1)^{10}(2x-4)^{k+1}}{10^{k+1}}}{\frac{k^{10}(2x-4)^k}{10^k}}\right|$

$$= \lim_{k\to\infty}\left|\frac{(k+1)^{10}(2x-4)^{k+1}}{10^{k+1}}\cdot\frac{10^k}{k^{10}(2x-4)^k}\right|$$

$$= \lim_{k\to\infty}\left|\frac{(k+1)^{10}(2x-4)}{10\,k^{10}}\right|$$

$$= \lim_{k\to\infty}\left|\frac{1}{10}\left(\frac{k+1}{k}\right)^{10}(2x-4)\right|$$

$$= \frac{1}{10}(1)^{10}|2x-4| = \left|\frac{2x-4}{10}\right|$$

We get convergence if $\left|\frac{2x-4}{10}\right| < 1$, that is, if

$$-1 < \frac{2x-4}{10} < 1 \implies -10 < 2x-4 < 10$$

$$\implies -6 < 2x < 14 \implies \boxed{-3 < x < 7}$$

Check endpoint $x=-3$: $\displaystyle\sum_{k=1}^{\infty}\frac{k^{10}(-10)^k}{10^k} = -1+2^{10}-3^{10}+4^{10}-5^{10}+\cdots$ ← Diverges

Check endpoint $x=7$: $\displaystyle\sum_{k=1}^{\infty}\frac{k^{10}(10)^k}{10^k} = \sum_{k=1}^{\infty} k^{10}$ ← Diverges.

Interval of convergence: $\boxed{(-3, 7)}$

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1. Find the interval of convergence for the following power series. Test endpoints (if any).

$$\sum_{k=1}^{\infty} \frac{k^{20}x^k}{(2k+1)!}$$

Ratio Test: $\displaystyle \lim_{k\to\infty} \frac{\frac{(k+1)^{20} x^{k+1}}{(2(k+1)+1)!}}{\frac{k^{20}x^k}{(2k+1)!}}$

$$= \lim_{k\to\infty} \frac{(k+1)^{20} x^{k+1}}{(2k+3)!} \cdot \frac{(2k+1)!}{k^{20} x^k}$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^{20} \frac{(2k+1)!}{(2k+3)(2k+2)(2k+1)!} x^k$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^{20} \frac{1}{(2k+3)(2k+1)} x^k$$

$$= 1^{20} \cdot 0 \cdot x^k = 0 < 1$$

Interval of convergence $\boxed{(-\infty, \infty)}$