Name: Richard

QUIZ 24 ♠

MATH 201
April 30, 2024

1. Use the Maclaurin series for $f(x) = e^x$ to obtain a series for the function $g(x) = x^2(e^x - 1)$. Write your final answer in sigma notation.

$$g(x) = x^2(e^x - 1) = x^2\left(\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right) - 1\right)$$

$$= x^2\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$$

$$= x^3 + \frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \frac{x^7}{5!} + \cdots$$

$$= \boxed{\sum_{k=1}^{\infty} \frac{x^{k+2}}{k!}}$$

2. Write the first four terms of the binomial series for $(x+1)^{-4}$.

$$(x+1)^{-4} = 1 - 4x + \frac{(-4)(-5)x^2}{2!} + \frac{(-4)(-5)(-6)}{3!}x^3 + \cdots$$

$$= \boxed{1 - 4x + 10x^2 - 20x^3 + \cdots}$$

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QUIZ 24 ◊

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April 30, 2024

1. Write the first four terms of the binomial series for $(x+1)^{-2}$.

$$(x+1)^{-2} = 1 + 2x + \frac{(-2)(-3)}{2!}x^2 + \frac{(-2)(-3)(-4)}{3!}x^3 + \cdots$$

$$= \boxed{1 - 2x + 3x^2 - 4x^3 + \cdots}$$

2. Use the Maclaurin series for $f(x) = e^x$ to obtain a series for the function $g(x) = e^{2x}$.
Write your final answer in sigma notation.

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \cdots$$

$$= 1 + 2x + \frac{2^2}{2!}x^2 + \frac{2^3}{3!}x^3 + \frac{2^4}{4!}x^4 +$$

$$= \boxed{\sum_{k=0}^{\infty} \frac{2^k}{k!} x^k}$$

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QUIZ 24 ♣

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1. Use the Maclaurin series for $f(x) = e^x$ to obtain a series for the function $g(x) = x^2(e^x - x - 1)$. Write your final answer in sigma notation.

$$g(x) = x^2(e^x - x - 1)$$

$$= x^2\left(\left(1 + x - \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right) - x - 1\right)$$

$$= x^2\left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$$

$$= \frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \frac{x^7}{5!} + \cdots$$

$$= \boxed{\sum_{k=2}^{\infty} \frac{x^{k+2}}{k!}}$$

2. Write the first four terms of the binomial series for $(x+1)^{-1}$.

$$(x+1)^{-1} = 1 - x + \frac{(-1)(-2)}{2!}x^2 + \frac{(-1)(-2)(-3)}{3!}x^3 + \cdots$$

$$= \boxed{1 - x + x^2 - x^3 + \cdots}$$

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QUIZ 24 ♡

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1. Write the first four terms of the binomial series for $(x+1)^{-3}$.

$$(x+1)^{-3} = 1 - 3x + \frac{(-3)(-4)}{2!}x^2 + \frac{(-3)(-4)(5)}{3!}x^3 + \cdots$$

$$= \boxed{1 - 3x + 6x^2 + 10x^3 + \cdots}$$

2. Use the Maclaurin series for $f(x) = e^x$ to obtain a series for the function $g(x) = xe^{-x}$.
Write your final answer in sigma notation.

$$g(x) = xe^{-x} = x\left(1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \frac{(-x)^4}{4!} + \frac{(-x)^5}{5!} + \cdots\right)$$

$$= x\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \cdots\right)$$

$$= x - x^2 + \frac{x^3}{2!} - \frac{x^4}{3!} + \frac{x^5}{4!} - \frac{x^6}{5!} + \cdots$$

$$= \frac{x^1}{0!} - \frac{x^2}{1!} + \frac{x^3}{2!} - \frac{x^4}{3!} + \frac{x^5}{4!} - \frac{x^6}{5!} + \cdots$$

$$= \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k!}}$$