Name: Richard

QUIZ 24

MATH 201
April 29, 2025

1. Use the Maclaurin series for $\cos(x)$ to find the series for $\cos\left(\sqrt{x}\right)$. Write your final answer in sigma notation.

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\cos(\sqrt{x}) = \sum_{k=0}^{\infty} (-1)^k \frac{\sqrt{x}^{\,2k}}{(2k)!} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\sqrt{x}^{\,2}\right)^k}{(2k)!}$$

$$= \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^k}{(2k)!}}$$

2. Find the first five terms of the binomial series for $\sqrt{1+x}$. $= (1+x)^{\frac{1}{2}}$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}x^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}x^3 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{4!}x^4 + \cdots$$

$$= \boxed{1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots}$$

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1. Use the Maclaurin series for $\sin(x)$ to find the series for $x\sin\left(x^2\right)$. Write your final answer in sigma notation.

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$x\sin(x^2) = x\left(\sum_{k=0}^{\infty} (-1)^k \frac{(x^2)^{2k+1}}{(2k+1)!}\right)$$

$$= x \sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+2}}{(2k+1)!}$$

$$= \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+3}}{(2k+1)!}}$$

2. Find the first five terms of the binomial series for $\sqrt[3]{1+x}$.

$$= (1+x)^{1/3}$$

$$= 1 + \frac{1}{3}x + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}x^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}x^3 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)}{4!}x^4 + \cdots$$

$$= \boxed{1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \frac{10}{243}x^4 + \cdots}$$