

1. Find the area contained between  $y = x^2 - 2x - 4$  and  $y = -x^2 + 4x + 4$  (Sketch the curves!)

$\uparrow$   $\uparrow$   
 $y$  intercept:  $-4$      $y$ -intercept:  $4$

Find the intersection points:

$$x^2 - 2x - 4 = -x^2 + 4x + 4$$

$$2x^2 - 6x - 8 = 0$$

$$2(x^2 - 3x - 4) = 0$$

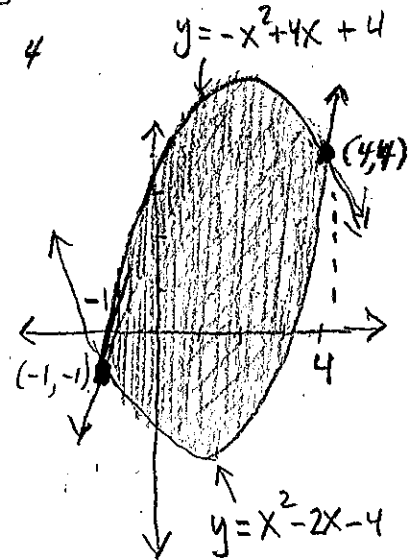
$$2(x+1)(x-4) = 0$$

$$\begin{matrix} \downarrow & \downarrow \\ x = -1 & x = 4 \end{matrix}$$

Intersections

$$(-1, (-1)^2 - 2(-1) - 4) = (-1, 1)$$

$$(4, 4^2 - 2 \cdot 4 - 4) = (4, 4)$$



$$A = \int_{-1}^4 (-x^2 + 4x + 4) - (x^2 - 2x - 4) dx = \int_{-1}^4 -2x^2 + 6x + 8 dx$$

$$= \left[ -2 \frac{x^3}{3} + 3x^2 + 8x \right]_{-1}^4 = \left( -2 \cdot \frac{4^3}{3} + 3 \cdot 4^2 + 8 \cdot 4 \right) - \left( -2 \frac{(-1)^3}{3} + 3(-1)^2 + 8(-1) \right)$$

$$= -\frac{128}{3} + 48 + 32 - \frac{2}{3} - 3 + 8 = -\frac{130}{3} + 85$$

$$= -\frac{130}{3} + \frac{255}{3} = \boxed{\frac{125}{3} \text{ square units}}$$

1. Find the area contained between  $y = 2\sqrt{x+1}$  and  $y = x+1$ .

(Sketch the curves!)

Find intersection points:

$$2\sqrt{x+1} = x+1$$

$$(2\sqrt{x+1})^2 = (x+1)^2$$

$$4(x+1) = x^2 + 2x + 1$$

$$4x + 4 = x^2 + 2x + 1$$

$$0 = x^2 - 2x - 3$$

$$0 = (x+1)(x-3)$$

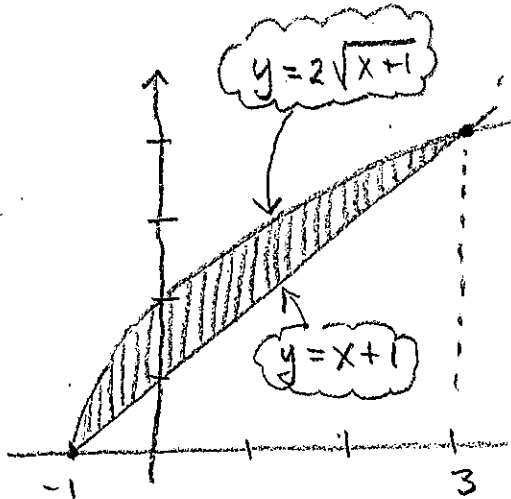
$$x = -1 \quad x = 3$$

$$y = -1 + 1 = 0 \quad y = 3 + 1 = 4$$

Intersections:

$$(-1, 0)$$

$$(3, 4)$$



$$A = \int_{-1}^3 (2\sqrt{x+1} - (x+1)) dx = \left[ \frac{2}{3} \sqrt{x+1}^3 - \frac{x^2}{2} - x \right]_{-1}^3$$

$$= \left( \frac{2}{3} \sqrt{-1+1}^3 - \frac{(-1)^2}{2} - (-1) \right) - \left( \frac{2}{3} \sqrt{3+1}^3 - \frac{3^2}{2} - 3 \right)$$

$$= \frac{2}{3} \cdot 0 - \frac{1}{2} + 1 - \frac{2}{3} \sqrt{4}^3 + \frac{9}{2} + 3$$

$$= \frac{1}{2} - \frac{16}{3} + \frac{9}{2} + 3 = \frac{3}{6} - \frac{32}{6} + \frac{27}{6} + \frac{18}{6} = \frac{16}{6}$$

$$= \boxed{\frac{8}{3} \text{ square units}}$$