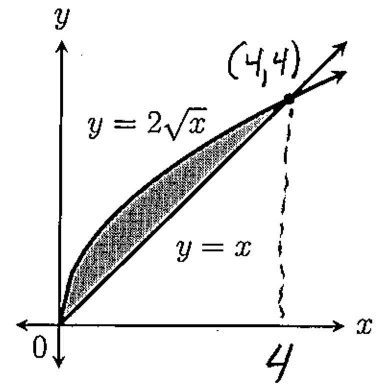
Name: Richard

QUIZ 3 ♠

MATH 201
January 25, 2024

1. The shaded region is rotated around the x-axis. Find the volume of the resulting solid.



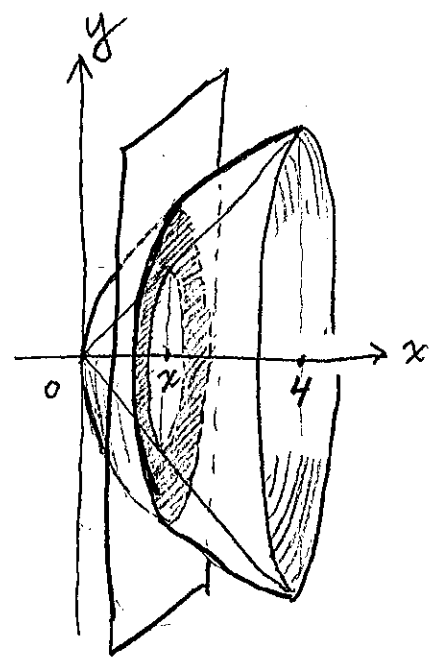
First, let's find the intersection of the two graphs. To do this solve $2\sqrt{x} = x \Rightarrow (2\sqrt{x})^2 = x^2$

$\Rightarrow 4x = x^2$

$\Rightarrow 4x - x^2 = 0$

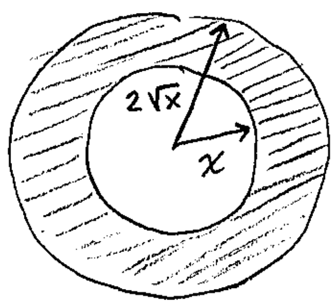
$\Rightarrow x(4-x) = 0$

$x = 0 \qquad x = 4$



Therefore they intersect at $(0,0)$ and $(4,4)$.

The cross-section at x is a washer with inner radius x and outer radius $2\sqrt{x}$. Therefore



$$\boxed{A(x) = \pi(2\sqrt{x})^2 - \pi x^2 = 4\pi x - \pi x^2}$$

$$V = \int_0^4 (4\pi x - \pi x^2)\,dx = \pi \int_0^4 4x - x^2\,dx =$$

$$\pi\left[2x^2 - \frac{x^3}{3}\right]_0^4 = \pi\left(\left(2 \cdot 4^2 - \frac{4^3}{3}\right) - \left(2 \cdot 0^2 - \frac{0^3}{3}\right)\right) =$$

$$= \pi\left(32 - \frac{64}{3}\right) = \pi\left(\frac{96}{3} - \frac{64}{3}\right) = \boxed{\frac{32\pi}{3} \text{ cubic units}}$$

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QUIZ 3 ◇

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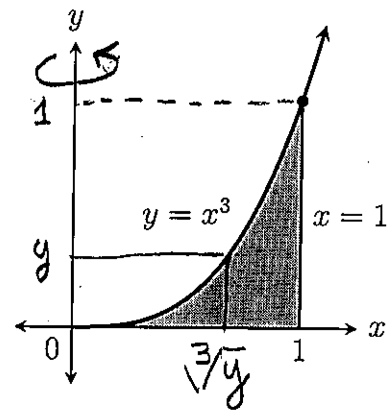
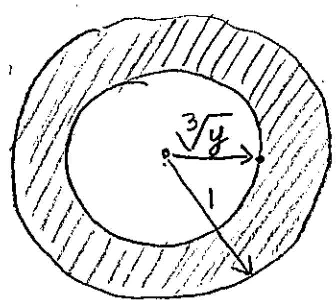
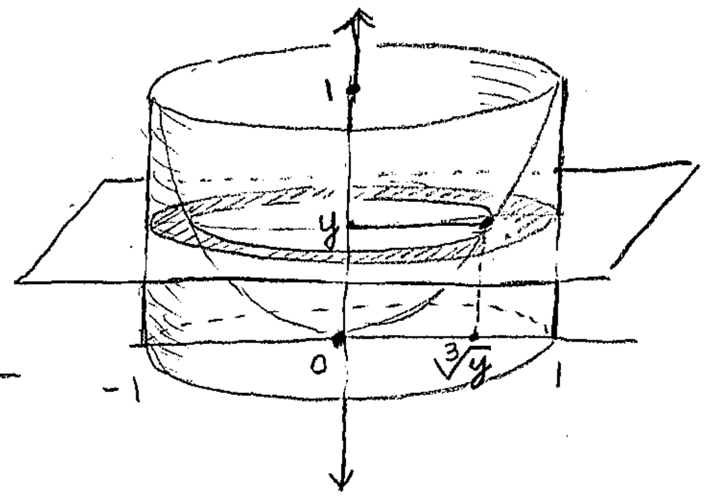
1. The shaded region is rotated around the y-axis. Find the volume of the resulting solid.

The cross-section at y is a washer with inner radius $\sqrt[3]{y}$ and outer radius 1.







Thus $A(y) = \pi \cdot 1^2 - \pi \sqrt[3]{y}^{\,2}$
$= \pi\left(1 - \sqrt[3]{y}^{\,2}\right)$

$$V = \int_0^1 \pi\left(1 - \sqrt[3]{y}^{\,2}\right) dy = \pi \int_0^1 \left(1 - y^{2/3}\right) dy$$

$$= \pi\left[y - \frac{y^{2/3+1}}{2/3+1}\right]_0^1 = \pi\left[y - \frac{3\sqrt[3]{y}^{\,5}}{5}\right]_0^1 =$$

$$= \pi\left(\left(1 - \frac{3\sqrt[3]{1}^{\,5}}{5}\right) - \left(0 - \frac{3\sqrt[3]{0}^{\,5}}{5}\right)\right)$$

$$= \pi\left(1 - \frac{3}{5}\right) = \boxed{\frac{2\pi}{5} \text{ cubic units}}$$

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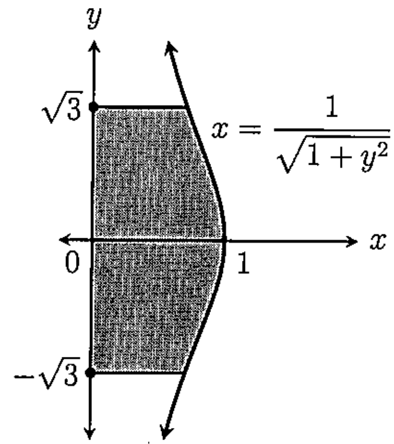
QUIZ 3 ♣

MATH 201
January 25, 2024

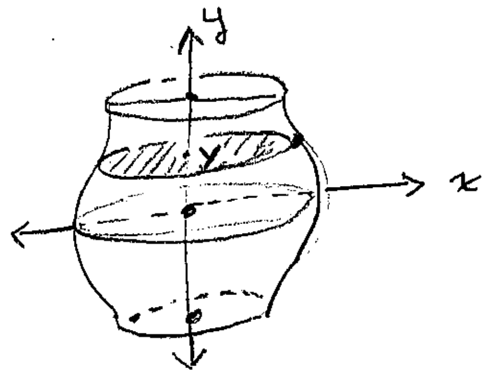
1. The shaded region is rotated around the y-axis. Find the volume of the resulting solid.



The cross-section at y is a circle of radius $r = \frac{1}{\sqrt{1+y^2}}$. Therefore

$$A(y) = \pi r^2 = \pi\left(\frac{1}{\sqrt{1+y^2}}\right)^2$$

$$A(y) = \frac{\pi}{1+y^2}$$

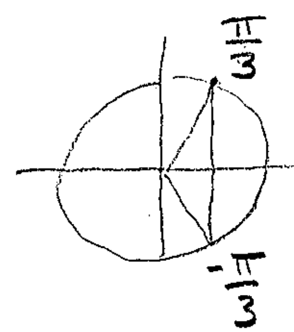


$$V = \int_{-\sqrt{3}}^{\sqrt{3}} \frac{\pi}{1+y^2}\,dy$$

$$= \Big[\pi \tan^{-1}(y)\Big]_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \pi \tan^{-1}(\sqrt{3}) - \pi \tan^{-1}(-\sqrt{3})$$

$$= \pi\frac{\pi}{3} - \left(\pi\left(-\frac{\pi}{3}\right)\right)$$



$$= \frac{\pi^2}{3} + \frac{\pi^2}{3} = \boxed{\frac{2\pi^2}{3} \text{ cubic units}}$$

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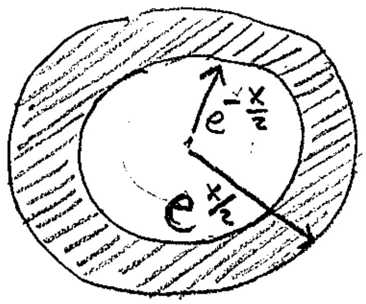
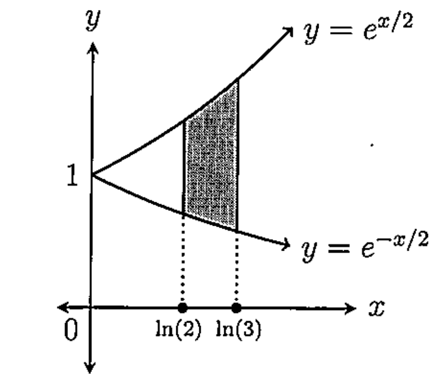
QUIZ 3 ♡

MATH 201

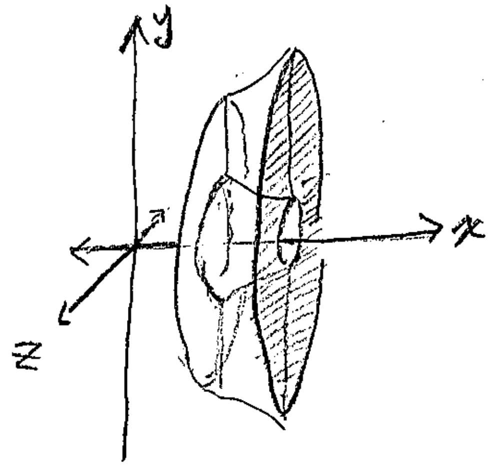
January 25, 2024

1. The shaded region is rotated around the x-axis. Find the volume of the resulting solid.





The cross-section at x is a washer with inner radius $e^{-x/2}$ and outer radius $e^{x/2}$. Therefore



$$A(x) = \pi\left(e^{x/2}\right)^2 - \pi\left(e^{-x/2}\right)^2$$

$$A(x) = \pi e^{x} - \pi e^{-x}$$

$$V = \int_{\ln(2)}^{\ln(3)} A(x)\,dx = \int_{\ln(2)}^{\ln(3)} (\pi e^{x} - \pi e^{-x})\,dx$$

$$= \pi\left[e^{x} + e^{-x}\right]_{\ln(2)}^{\ln(3)} = \pi\left(\left(e^{\ln(3)} + e^{-\ln(3)}\right) - \left(e^{\ln(2)} + e^{-\ln(2)}\right)\right)$$

$$= \pi\left(\left(3 + e^{\ln(\frac{1}{3})}\right) - \left(2 + e^{\ln(1/2)}\right)\right)$$

$$= \pi\left(\left(3 + \frac{1}{3}\right) - \left(2 + \frac{1}{2}\right)\right) = \pi\left(1 + \frac{1}{3} - \frac{1}{2}\right) = \pi\left(\frac{6}{6} + \frac{2}{6} - \frac{3}{6}\right)$$

$$= \boxed{\frac{5\pi}{6} \text{ cubic units}}$$