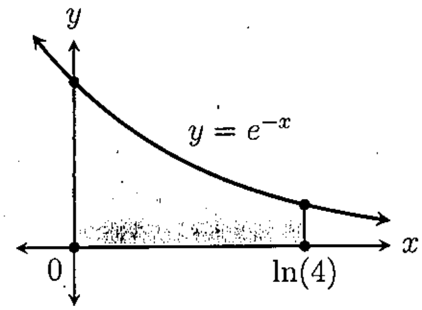
Name: Richard

QUIZ 3

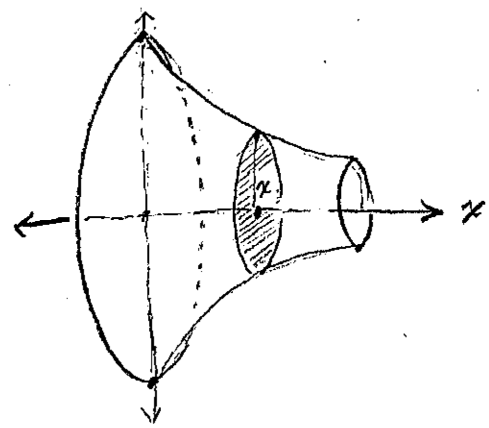
MATH 201
January 23, 2025

1. The shaded region is rotated around the $x$-axis. Find the volume of the resulting solid.




The cross-section at $x$ is a circle of radius $e^{-x}$ with area

$$A(x) = \pi(e^{-x})^2 = \pi e^{-2x}$$




Therefore the volume is

$$V = \int_0^{\ln 4} \pi e^{-2x}\, dx$$

$$= \pi \int_0^{\ln 4} e^{-2x}\, dx$$

$$= \pi \left[-\frac{1}{2} e^{-2x}\right]_0^{\ln 4} = \pi\left(-\frac{1}{2} e^{-2\ln 4} - \left(-\frac{1}{2} e^{-2\cdot 0}\right)\right)$$

$$= \pi\left(-\frac{1}{2} e^{\ln 4^{-2}} + \frac{1}{2} e^{0}\right) = \pi\left(-\frac{4^{-2}}{2} + \frac{1}{2}\right)$$

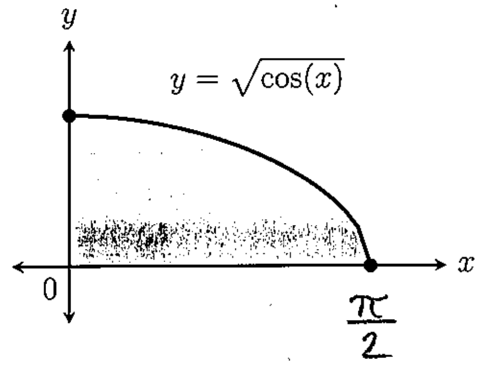
$$= \pi\left(-\frac{1}{32} + \frac{1}{2}\right) = \pi\left(-\frac{1}{32} + \frac{16}{32}\right) = \boxed{\frac{15\pi}{32} \text{ cubic units}}$$

Name: Richard

QUIZ 3

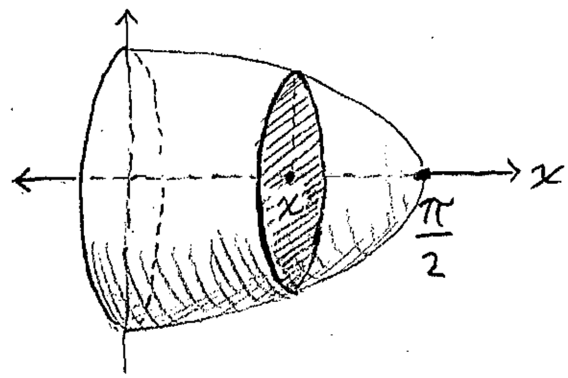
MATH 201
January 25, 2025

1. The shaded region is rotated around the $x$-axis. Find the volume of the resulting solid.




The cross-section at $x$ is a circle of radius $\sqrt{\cos(x)}$ with area $A(x) = \pi \sqrt{\cos(x)}^2$

i.e. $A(x) = \pi \cos(x)$




Therefore the volume is

$$\int_0^{\pi/2} A(x)\,dx = \int_0^{\pi/2} \pi \cos(x)\,dx$$

$$= \Big[\pi \sin(x)\Big]_0^{\pi/2} = \pi \sin\left(\frac{\pi}{2}\right) - \pi \sin(0)$$

$$= \pi \cdot 1 - \pi \cdot 0 = \boxed{\pi \text{ cubic units}}$$