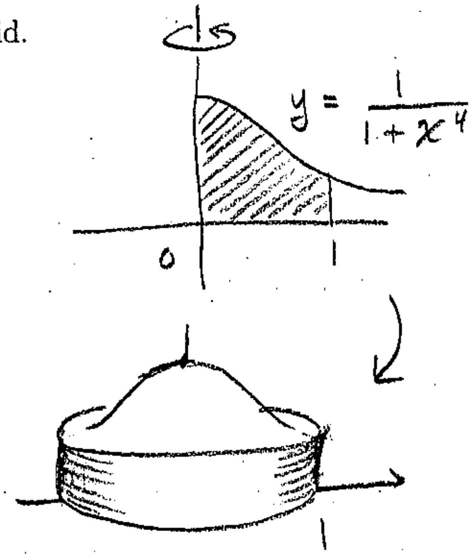
Name: Richard

QUIZ 4

MATH 201
January 28, 2025

1. The region enclosed by $y = \dfrac{1}{1+x^4}$, $x = 0$, $x = 1$ and $y = 0$ is rotated about the y-axis. Use the shell method to find the volume of the resulting solid.



$$V = \int_0^1 2\pi x \frac{1}{1+x^4}\, dx$$

$$= \pi \int_0^1 \frac{1}{1+(x^2)^2}\, 2x\, dx$$

$u = x^2$
$du = 2x\, dx$

$$= \pi \int_{0^2}^{1^2} \frac{1}{1+u^2}\, du$$

$$= \pi \Big[\tan^{-1}(u)\Big]_0^1$$

$$= \pi\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

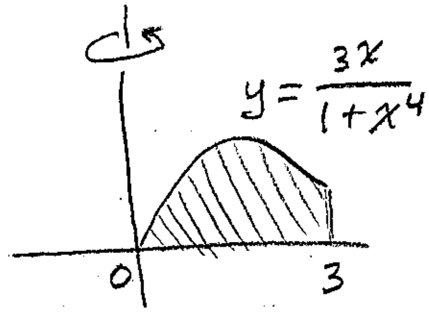
$$= \pi\left(\frac{\pi}{4} - 0\right) = \boxed{\frac{\pi^2}{4} \text{ cubic units}}$$

Name: Richard

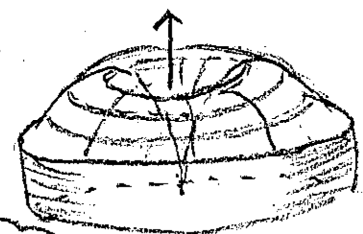
QUIZ 4

MATH 201
January 28, 2025

1. The region enclosed by $y = \dfrac{3x}{1+x^3}$, $x = 0$, $x = 3$ and $y = 0$ is rotated about the y-axis. Use the shell method to find the volume of the resulting solid.



$$V = \int_0^3 2\pi x \frac{3x}{1+x^3}\,dx$$



$$= 2\pi \int_0^3 \frac{3x^2}{1+x^3}\,dx$$

$u = 1+x^3$
$du = 3x^2\,dx$

$$= 2\pi \int_{1+0^3}^{1+3^3} \frac{1}{u}\,du$$

$$= 2\pi \Big[\ln u\Big]_1^{28}$$

$$= 2\pi\big(\ln(28) - \ln(1)\big)$$

$$= \boxed{2\pi \ln(28) \text{ cubic units}}$$