Name: Richard

Quiz 5

MATH 201
January 30, 2025

1. Find the length of the curve $y = \frac{1}{3}x^{3/2} - x^{1/2}$ on the interval $[1, 4]$.

$$L = \int_1^4 \sqrt{1 + (f'(x))^2}\, dx = \int_1^4 \sqrt{1 + \left(\frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}\right)^2}\, dx$$

$$= \int_1^4 \sqrt{1 + \left(\frac{1}{2}x^{1/2}\right)^2 - \frac{1}{2}x^{1/2}\frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-1/2}\frac{1}{2}x^{1/2} + \left(-\frac{1}{2}x^{-1/2}\right)^2}\, dx$$

$$= \int_1^4 \sqrt{1 + \frac{x}{4} - \frac{1}{2} + \frac{1}{4x}}\, dx$$

$$= \int_1^4 \sqrt{\frac{x}{4} + \frac{1}{2} + \frac{1}{4x}}\, dx = \int_1^4 \sqrt{\left(\frac{\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}\right)^2}\, dx$$

$$= \int_1^4 \frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2}\, dx = \left[\frac{1}{3}x^{3/2} + x^{\frac{1}{2}}\right]_1^4$$

$$= \left[\frac{1}{3}\sqrt{x}^3 + \sqrt{x}\right]_1^4 = \left(\frac{1}{3}\sqrt{4}^3 + \sqrt{4}\right) - \left(\frac{1}{3}\sqrt{1} + \sqrt{1}\right)$$

$$= \frac{8}{3} + 2 - \frac{1}{3} - 1 = \frac{7}{3} + 1 = \boxed{\frac{10}{3} \text{ units}}$$

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QUIZ 5

MATH 201

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1. Find the length of the curve $y = \frac{1}{2}\left(e^x + e^{-x}\right)$ on the interval $[-1, 1]$.

$$L = \int_{-1}^{1} \sqrt{1 + (y')^2}\, dx = \int_{-1}^{1} \sqrt{1 + \left(\tfrac{1}{2}(e^x - e^{-x})\right)^2}\, dx$$

$$= \int_{-1}^{1} \sqrt{1 + \tfrac{1}{4}\left((e^x)^2 - 2e^x e^{-x} + (e^{-x})^2\right)}\, dx$$

$$= \int_{-1}^{1} \sqrt{1 + \tfrac{1}{4}\left((e^x)^2 - 2 + (e^{-x})^2\right)}\, dx$$

$$= \int_{-1}^{1} \sqrt{\frac{4 + (e^x)^2 - 2 + (e^{-x})^2}{4}}\, dx$$

$$= \int_{-1}^{1} \frac{\sqrt{(e^x)^2 + 2 + (e^{-x})^2}}{2}\, dx$$

$$= \frac{1}{2}\int_{-1}^{1} \sqrt{(e^x + e^{-x})^2}\, dx = \frac{1}{2}\int_{-1}^{1} e^x + e^{-x}\, dx$$

$$= \frac{1}{2}\left[e^x - e^{-x}\right]_{-1}^{1} = \frac{1}{2}\left((e^1 - e^{-1}) - (e^{-1} - e^1)\right)$$

$$= \frac{1}{2}\left(2e - \frac{2}{e}\right) = \boxed{e - \frac{1}{e} \text{ units}}$$