Name: Richard

QUIZ 6

MATH 201
February 4, 2025

1. The curve $y = \sqrt{4x+6}$ for $1 \le x \le 5$ is rotated around the x-axis.
Find the area of the resulting surface.

$$A = \int_1^5 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

$$= \int_1^5 2\pi\sqrt{4x+6}\sqrt{1+\left(\frac{4}{2\sqrt{4x+6}}\right)^2}\,dx$$

$u = 4x+10$
$du = 4dx$

$$= \int_1^5 2\pi\sqrt{4x+6}\sqrt{1+\frac{4}{4x+6}}\,dx$$

$$= \int_1^5 2\pi\sqrt{4x+6}\sqrt{\frac{4x+10}{4x+6}}\,dx$$

$$= \int_1^5 2\pi\,\cancel{\sqrt{4x+6}}\,\frac{\sqrt{4x+10}}{\cancel{\sqrt{4x+6}}}\,dx = \pi\int_1^5 \sqrt{4x+10}\;2\,dx$$

$$= \frac{\pi}{2}\int_1^5 \sqrt{4x+10}\;4\,dx = \frac{\pi}{2}\int_{4\cdot 1+10}^{4\cdot 5+10}\sqrt{u}\,du$$

$$= \frac{\pi}{2}\left[\frac{2\sqrt{u}^3}{3}\right]_{14}^{30} = \frac{\pi}{2}\left(\frac{2\sqrt{30}^3}{3} - \frac{2\sqrt{14}^3}{3}\right)$$

$$= \frac{\pi}{6}\left(60\sqrt{30} - 28\sqrt{14}\right) = \boxed{\frac{\pi}{3}\left(30\sqrt{30} - 14\sqrt{14}\right) \text{ square units}}$$

$$= \boxed{\frac{2\pi}{3}\left(15\sqrt{30} - 7\sqrt{14}\right) \text{ square units}}$$

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Quiz 6

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1. The curve $y = \sqrt{5x - x^2}$ for $1 \le x \le 4$ is rotated around the x-axis.
Find the area of the resulting surface.

$$A = \int_1^4 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_1^4 2\pi \sqrt{5x - x^2} \sqrt{1 + \left(\frac{5 - 2x}{2\sqrt{5x - x^2}}\right)^2}\, dx$$

$$= 2\pi \int_1^4 \sqrt{5x - x^2} \sqrt{1 + \frac{25 - 20x + 4x^2}{4(5x - x^2)}}\, dx$$

$$= 2\pi \int_1^4 \sqrt{5x - x^2} \sqrt{\frac{20x - 4x^2 + 25 - 20x + 4x^2}{4(5x - x^2)}}\, dx$$

$$= \cancel{2}\pi \int_1^4 \cancel{\sqrt{5x - x^2}}\ \frac{\sqrt{25}}{\cancel{2}\cancel{\sqrt{5x - x^2}}}\, dx$$

$$= \pi \int_1^4 5\, dx = \pi \Big[5x \Big]_1^4 = \pi (5 \cdot 4 - 5 \cdot 1)$$

$$= \boxed{15\pi \text{ square units}}$$