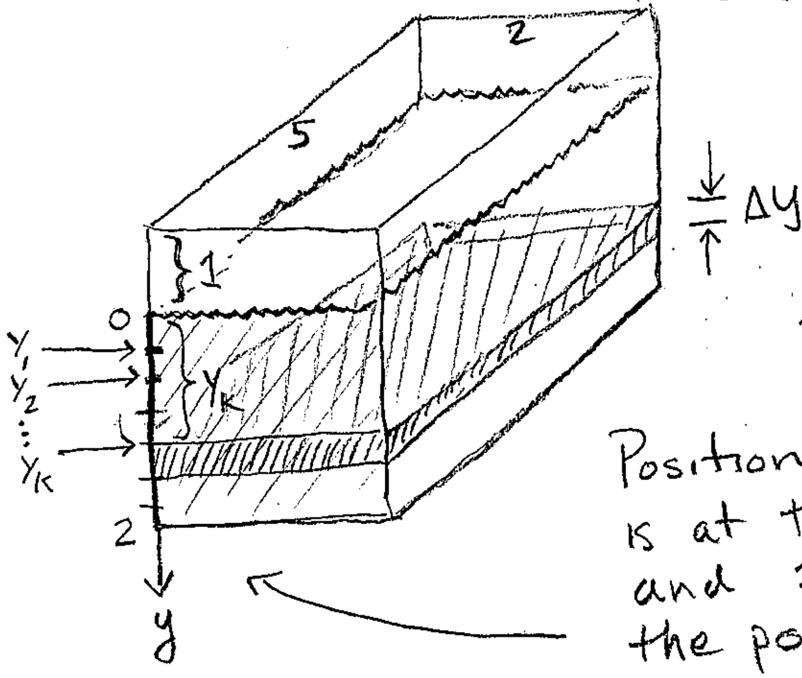
Name: Richard

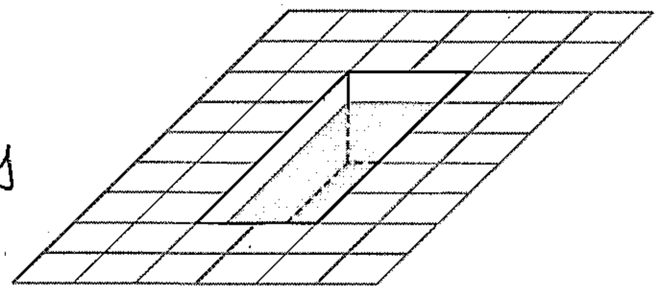
Quiz 7 ♠

MATH 201

February 8, 2024

1. A swimming pool is 5 meters long, 2 meters wide, and 3 meters deep. It is filled with water to a depth of 2 meters. (So the water level is 1 meter below ground level.) How much work is required to pump all the water to ground level? (Once pumped out, water does not flow back into the pool!)





Position the y-axis so that 0 is at the original water level and 2 is at the bottom of the pool. Let $\Delta y = \frac{2-0}{n} = \frac{2}{n}$ and $y_k = k\Delta x = \frac{2k}{n}$ for $k = 1, 2, \dots, n$.

Divide the water into layers, each a box of dimensions $2 \times 5 \times \Delta y$, and the top of layer k is at $y = y_k$, as shown above.

Mass of layer k: $1000 \cdot 5 \cdot 2 \cdot \Delta y = 10000\,\Delta y$ kg.

From picture, layer k must be pumped up a distance of $1 + y_k$ meters.

Work done in lifting layer k: $W_k \approx mad$
$= 10000\,\Delta y\; 9.8\,(1+y_k)$
$= 98000\,(1+y_k)\,\Delta y$

$\therefore$ Total work done is $\lim_{n\to\infty} \sum_{k=1}^{n} 98000(1+y_k)\Delta y$

$$= \int_0^2 98000(1+y)\,dy = 98000\left[y + \frac{y^2}{2}\right]_0^2 = 98000\left(2 + \frac{2^2}{2}\right) = 98000 \cdot 4$$

$= \boxed{392000\ \text{J}}$

Relevant Facts:
The density of water is 1000 kg per cubic meter.
Acceleration due to gravity: 9.8 meters per second per second.

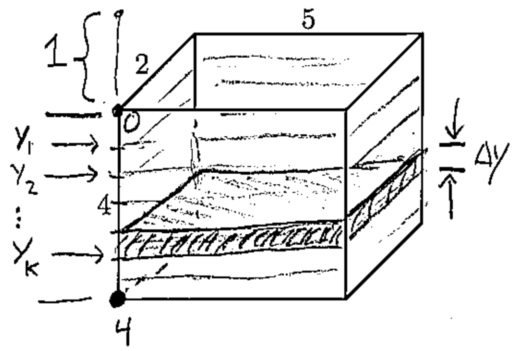
Name: Richard

QUIZ 7 ◊

MATH 201
February 8, 2024

1. A box-shaped tank is 5 meters long, 2 meters wide and 4 meters tall, as shown below. The tank is filled to the top with water. How much work is required to pump all the water to a height of 1 meter above the top of the tank?

Put the y axis so that 0 is at the top of the tank and 4 is at the bottom →



Divide the water into layers, so that each layer has height $\Delta y = \frac{4-2}{n}$ and layer k is at x_k on the y-axis.

Then layer #k must be pumped a distance of approx. $1+y_k$ meters.

Volume of layer #k: $2 \cdot 5 \cdot \Delta y = 10\Delta y$.

Mass of layer #k: $1000 \cdot 10\Delta y = 10000\Delta y$

Work done in lifting layer k: $W_k = mad$

$= 10000\Delta y \, 9.8(1+y_k)$

$= 98000(1+y_k)\Delta y$ J

Total work done: $W =$

$$\lim_{n\to\infty} \sum_{k=1}^{n} 98000(1+y_k)\Delta y = \int_0^4 98000(1+y)\,dy$$

$$= 98000\left[y + \frac{y^2}{2}\right]_0^4 = 98000\left[4 + \frac{4^2}{2}\right] = 98000 \cdot 12$$

$$= \boxed{1176000 \text{ J}}$$

$$\begin{array}{r} 98000 \\ \times\ 12 \\ \hline 196000 \\ 98000\ \\ \hline 1176000 \end{array}$$

Relevant Facts:
The density of water is 1000 kg per cubic meter.
Acceleration due to gravity: 9.8 meters per second per second.

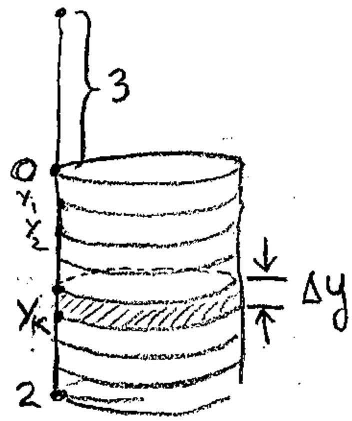
Name: Richard

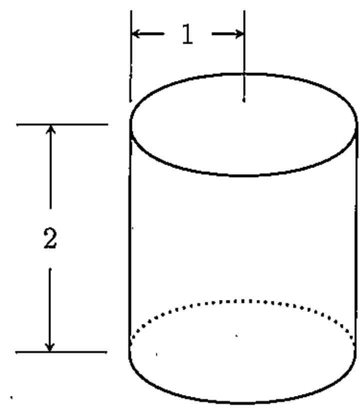
QUIZ 7 ♣

MATH 201
February 8, 2024

1. A cylindrical tank of radius 1 meter and height 2 meters is filled to the top with water. How much work is required to pump all the water to a height of 3 meters above the top of the tank?

Orient the y-axis so that 0 is at the top of the tank and 2 is at the bottom. Divide this interval into n parts in the usual way, so each subinterval has width Δy.





Divide the water into layers, each of height Δy, so layer # K is at x_k, as illustrated.

Each layer has mass $1000\pi r^2 \Delta y = 1000\pi \Delta y$. And layer k must be pumped a distance of $3 + y_k$ so the work done in lifting layer k is

$$W_k \approx m a d = 1000\pi \Delta y \, 9.8 \, (3 + y_k) = 9800\pi (3 + y_k) \Delta y$$

The total work done is therefore

$$W = \lim_{n \to \infty} \sum_{k=1}^{n} 9800\pi (3 + y_k) \Delta y$$

$$= \int_0^2 9800\pi (3+y)\,dy = 9800\pi \left[3y + \frac{y^2}{2} \right]_0^2$$

$$= 9800\pi \left(3 \cdot 2 + \frac{2^2}{2} \right) = \boxed{78400\pi \text{ J}}$$

6
9800
8
78400

Relevant Facts:
The density of water is 1000 kg per cubic meter.
Acceleration due to gravity: 9.8 meters per second per second.

Name: Richard

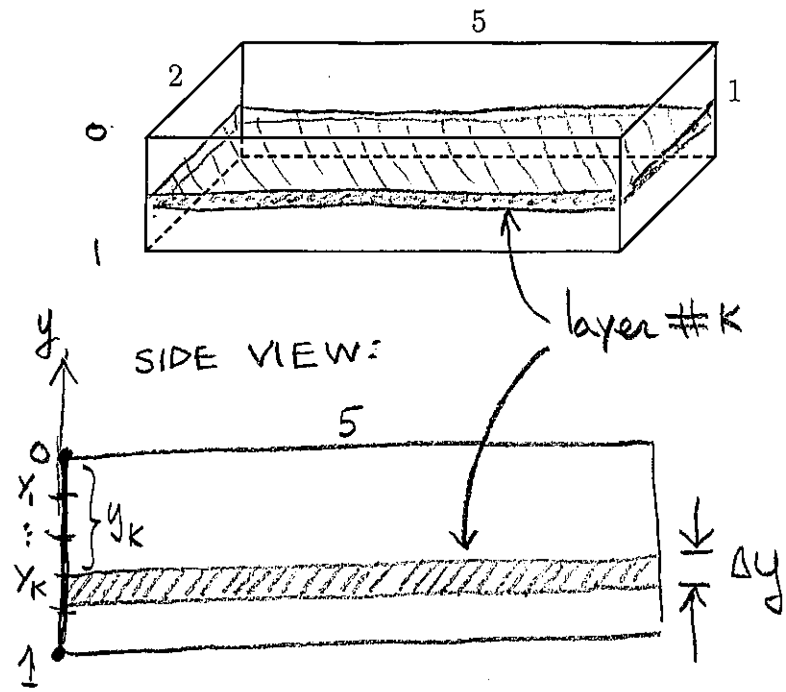
QUIZ 7 ♡

MATH 201
February 8, 2024

1. A swimming pool, completely full of water, is 5 meters long and 2 meters wide and 1 meter deep. How much work is required to pump all the water to ground level? (Once pumped out, water does not flow back into the pool!)

Align the y axis so that 0 is at the top of the pool and 1 is at the bottom.

Let $\Delta y = \frac{1-0}{n} = \frac{1}{n}$ and put layer #K at $y_k = k\Delta x$, as illustrated →



Therefore layer #K must be pumped a distance of y_k m.

Mass of layer #K $= 1000\, l w h = 1000\ 5 \cdot 2 \cdot \Delta y$
$= 10000\, \Delta y$.

Work done in pumping layer K: $W_k = m\, a\, d$
$= 10000\, \Delta y\ 9.8\ y_k$
$= 98000\, y_k\, \Delta y$

Total work done is thus

$$W = \lim_{n \to \infty} \sum_{k=1}^{n} 98000\, y_k\, \Delta y = \int_0^1 98000\, y\, dy$$

$$= 98000 \left[\frac{y^2}{2}\right]_0^1 = 98000 \cdot \frac{1}{2} = \boxed{49000\ \mathrm{J}}$$

Relevant Facts:
The density of water is 1000 kg per cubic meter.
Acceleration due to gravity: 9.8 meters per second per second.