

$$1. \int \tan\left(\frac{x}{2}\right) dx = \frac{1}{\frac{1}{2}} \ln|\sec\left(\frac{x}{2}\right)| + C = \boxed{2 \ln|\sec\left(\frac{x}{2}\right)| + C}$$

$$2. \int \frac{x^3 - 2}{x + 1} dx = \int x^2 - x + 1 - \frac{3}{x + 1} dx = \boxed{\frac{x^3}{3} - \frac{x^2}{2} + x - 3 \ln|x + 1| + C}$$

$$\begin{array}{r} x^2 - x + 1 \\ x + 1 \overline{) x^3 + 0x^2 + 0x - 2} \\ \underline{x^3 + x^2} \\ -x^2 + 0x \\ \underline{-x^2 - x} \\ x - 2 \\ \underline{x + 1} \\ -3 \end{array}$$

$$1. \int \sec(\pi x) dx = \boxed{\frac{1}{\pi} \ln|\sec(\pi x) + \tan(\pi x)| + C}$$

$$2. \int \frac{x}{x^4 + 2x^2 + 1} dx = \int \frac{x}{(x^2 + 1)^2} dx = \int \frac{1}{u^2} \cdot \frac{1}{2} du = \frac{1}{2} \int u^{-2} du$$

$$\begin{array}{l} u = x^2 + 1 \\ du = 2x dx \\ \frac{1}{2} du = x dx \end{array}$$

$$= \frac{1}{2} \frac{u^{-1}}{-1} + C = \frac{-1}{2u} + C$$

$$= \frac{-1}{2(x^2 + 1)} + C$$

$$= \boxed{\frac{-1}{2x^2 + 2} + C}$$

$$1. \int \cot(\pi x) dx = \boxed{\frac{1}{\pi} \ln |\sin(\pi x)| + C}$$

$$2. \int \frac{1}{x^2 + 2x + 5} dx = \int \frac{1}{x^2 + 2x + 1 + 4} dx = \int \frac{1}{(x+1)^2 + 2^2} dx$$

$$\begin{array}{l} u = x+1 \\ du = dx \end{array}$$

$$= \int \frac{1}{u^2 + 2^2} du = \frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right) + C$$

$$= \boxed{\frac{1}{2} \tan^{-1}\left(\frac{x+1}{2}\right) + C}$$

$$1. \int \csc(5x) dx = \boxed{-\frac{1}{5} \ln |\csc(5x) + \cot(5x)| + C}$$

$$2. \int \frac{dx}{x^{-1} + 1} = \int \frac{dx}{\frac{1}{x} + 1} = \int \frac{dx}{\frac{1+x}{x}} = \int \frac{x}{1+x} dx$$

$$= \int \frac{u-1}{u} du = \int 1 - \frac{1}{u} du$$

$$\begin{cases} u = 1+x & x = u-1 \\ du = dx \end{cases}$$

$$= u - \ln|u| + C$$

$$= 1+x - \ln|1+x| + C = \boxed{x - \ln|1+x| + C}$$

The 1 and C combine into a single C