Name: Richard

QUIZ 9

MATH 201
February 20, 2025

1. $\int x^2 \sin(3x)\,dx =$ $x^2\left(-\frac{1}{3}\cos(3x)\right) - \int -\frac{1}{3}\cos(3x)\,2x\,dx$

$u = x^2 \quad dv = \sin(3x)\,dx$

$du = 2x\,dx \quad v = \frac{-1}{3}\cos(3x)$

$= -\frac{x^2\cos(3x)}{3} + \frac{2}{3}\int x\cos(3x)\,dx$

$= -\frac{x^2\cos(3x)}{3} + \frac{2}{3}\left(\frac{x\sin(3x)}{3} - \int \frac{1}{3}\sin(3x)\,dx\right)$

Integration by parts again!

$u = x \quad dv = \cos(3x)\,dx$

$du = dx \quad v = \frac{1}{3}\sin(3x)$

$= \boxed{-\frac{x^2\cos(3x)}{3} + \frac{2x\sin(3x)}{9} + \frac{2\cos(3x)}{27} + C}$

Check: $\frac{d}{dx}\left[-\frac{x^2\cos(3x)}{3} + \frac{2x\sin(3x)}{9} + \frac{2\cos(3x)}{27} + C\right]$

$= \frac{-2x\cos(3x) + x^2 3\sin(3x)}{3} + \frac{2\sin(3x) + 6x\cos(3x)}{9} - \frac{6\sin(3x)}{27}$

$= -\frac{2}{3}x\cos(3x) + x^2\sin(3x) + \frac{2}{9}\sin(3x) + \frac{2}{3}x\cos(3x) - \frac{2}{9}\sin(3x)$

$= x^2\sin(3x)$ YES

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1. $\int x\sin(x)\cos(x)\,dx =$

$u = x \qquad dv = \sin(x)\cos(x)\,dx$

$du = dx \qquad v = \int \sin(x)\cos(x)\,dx = \frac{\sin^2(x)}{2}$

$= \frac{x\sin^2(x)}{2} - \int \frac{\sin^2(x)}{2}\,du$

$= \frac{x\sin^2(x)}{2} - \frac{1}{2}\int \sin^2(x)\,dx$ Integration by Parts

$= \frac{x\sin^2(x)}{2} - \frac{1}{2}\cdot\frac{1}{2}\left(x - \sin(x)\cos(x)\right) + C$

$= \boxed{\frac{x\sin^2(x)}{2} - \frac{x}{4} + \frac{\sin(x)\cos(x)}{4} + C}$

Check: $\frac{d}{dx}\left[\frac{x\sin^2(x)}{2} - \frac{x}{4} + \frac{\sin(x)\cos(x)}{4} + C\right]$

$= \frac{\sin^2(x) + 2x\sin(x)\cos(x)}{2} - \frac{1}{4} + \frac{\cos^2(x) - \sin^2(x)}{4}$

$= \frac{\sin^2(x)}{2} + x\sin(x)\cos(x) + \frac{-1 + \cos^2(x) - \sin^2(x)}{4}$

$= \frac{\sin^2(x)}{2} + x\sin(x)\cos(x) + \frac{-\sin^2(x) - \sin^2(x)}{4}$ ($-1 + \cos^2(x) = -\sin^2(x)$)

$= \frac{\sin^2(x)}{2} + x\sin(x)\cos(x) - \frac{\sin^2(x)}{2} = x\sin(x)\cos(x)$ ✓ YES