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TEST 1
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MATH 201
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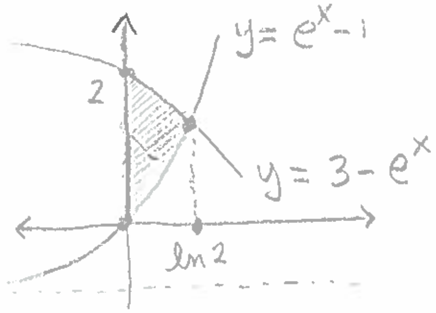
1. Find the area of the region in the first quadrant, between the graphs of $y = e^x - 1$ and $y = 3 - e^x$.



Find intersection $e^x - 1 = 3 - e^x$

$$2e^x = 4$$

$$e^x = 2 \implies \boxed{x = \ln 2}$$

$$A = \int_0^{\ln 2} (3 - e^x) - (e^x - 1)\,dx = \int_0^{\ln 2} 4 - 2e^x\,dx$$

$$= \Big[4x - 2e^x\Big]_0^{\ln 2} = \left(4\ln 2 - 2e^{\ln 2}\right) - \left(4\cdot 0 - 2e^0\right) = (4\ln 2 - 2\cdot 2) - (-2)$$

$$= \boxed{\ln 16 - 2 \text{ square units}}$$

2. The curve $y = 2\sqrt{x}$ for $0 \le x \le 2$ is rotated around the x-axis. Find the area of the resulting surface.

$$A = \int_0^2 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx = 2\pi \int_0^2 2\sqrt{x}\sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\,dx$$

$$= 4\pi \int_0^2 \sqrt{x}\sqrt{1 + \frac{1}{x}}\,dx = 4\pi \int_0^2 \sqrt{x\left(1 + \frac{1}{x}\right)}\,dx$$

$$= 4\pi \int_0^2 \sqrt{x+1}\,dx = 4\pi \left[\frac{2\sqrt{x+1}^{\,3}}{3}\right]_0^2$$

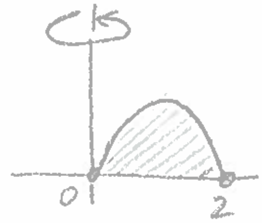
$$= 4\pi\left(\frac{2\sqrt{2+1}^{\,3}}{3} - \frac{2\sqrt{0+1}^{\,3}}{3}\right) = 8\pi\left(\frac{\sqrt{3}^{\,3}}{3} - \frac{1}{3}\right)$$

$$= \boxed{\frac{8\pi}{3}\left(3\sqrt{3} - 1\right) \text{ square units}}$$

3. Consider the region between the graph of $y = 2x - x^2$ and the x-axis.
This region is revolved around the y-axis. What is the volume of the resulting solid?

$y = 2x - x^2 = x(2-x)$

x-intercepts $(0 \quad 2)$



Volume by shells: $V = \int_0^2 2\pi x f(x)\,dx =$

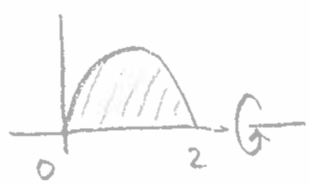
$$= \int_0^2 2\pi x(2x - x^2)\,dx = 2\pi \int_0^2 2x^2 - x^3\,dx$$

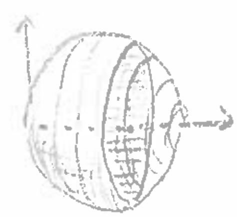
$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = 2\pi\left(\frac{2\cdot 2^3}{3} - \frac{2^4}{4}\right) = 2\pi\left(\frac{16}{3} - \frac{16}{4}\right)$$

$$= 32\pi\left(\frac{1}{3} - \frac{1}{4}\right) = 32\pi\left(\frac{4}{12} - \frac{3}{12}\right) = \frac{32\pi}{12} = \boxed{\frac{8\pi}{3} \text{ cubic units}}$$

4. Consider the region between the graph of $y = 2x - x^2$ and the x-axis. (Same region as above.)
This region is revolved around the x-axis. What is the volume of the resulting solid?

Volume by slicing





$$V = \int_0^2 \pi(2x - x^2)^2\,dx = \pi\int_0^2 (4x^2 - 4x^3 + x^4)\,dx$$

$$= \pi\left[4\frac{x^3}{3} - 4\frac{x^4}{4} + \frac{x^5}{5}\right]_0^2 = \pi\left(4\frac{2^3}{3} - 2^4 + \frac{2^5}{5}\right)$$

$$= \pi\left(\frac{32}{3} - 16 + \frac{32}{5}\right) = 16\pi\left(\frac{2}{3} - 1 + \frac{2}{5}\right)$$

$$= 16\pi\left(\frac{10}{15} - \frac{15}{15} + \frac{6}{15}\right) = \boxed{\frac{16}{15}\pi \text{ cubic units}}$$

5. Find the arc length of the curve $y = \frac{2}{3}\left(x^2+1\right)^{3/2}$ from $x=0$ to $x=3$.

$$L = \int_0^3 \sqrt{1+(y')^2}\,dx = \int_0^3 \sqrt{1+\left(\frac{2}{3}\frac{3}{2}(x^2+1)^{1/2}\,2x\right)^2}\,dx$$

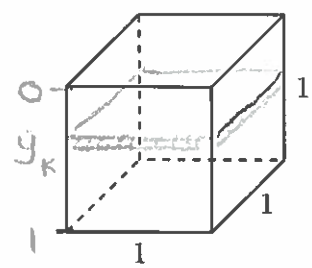
$$= \int_0^3 \sqrt{1+(x^2+1)\,4x^2}\,dx = \int_0^1 \sqrt{1+4x^4+4x^2}\,dx$$

$$= \int_0^3 \sqrt{4x^4+4x^2+1}\,dx = \int_0^3 \sqrt{(2x^2+1)^2}\,dx$$

$$= \int_0^3 2x^2+1\,dx = \left[\frac{2x^3}{3}+x\right]_0^3 = \frac{2\cdot 3^3}{3}+3 = \boxed{21 \text{ units}}$$

6. A cubic tank with sides of length 1 meter, is filled with water. Calculate the work required to pump all the water to the top of the tank. (Recall that the density of water is 1000 kilograms per cubic meter, and the acceleration due to gravity is 9.8 meters per second per second.)

Layer k has a volume of $1 \cdot 1 \cdot \Delta y$ m^3, So its mass is $1000\,\Delta y$ m^3.



Layer k must be pumped a distance of y_k meters, and the work done for this is mad $= 1000\,\Delta y\, 9.8\, y_k = 9800\, y_k\, \Delta y$ J

Total work done is $\lim_{n\to\infty} \sum_{k=1}^{n} 9800\, y_k\, \Delta y$

$$= \int_0^1 9800\, y\, dy = \left[9800\,\frac{y^2}{2}\right]_0^1$$

$$= \boxed{4900 \text{ J}}$$