

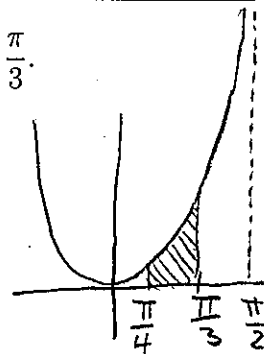
1. Find the area of the region bounded by $y = \tan^2(x)$, $y = 0$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$.

$$A = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2(x) dx = \left[\tan(x) - x \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \left(\tan\left(\frac{\pi}{3}\right) - \frac{\pi}{3} \right) - \left(\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} \right)$$

$$= \frac{\sqrt{3}}{2} - \frac{\pi}{3} - \frac{\sqrt{2}}{\sqrt{2}} + \frac{\pi}{4} = \sqrt{3} - 1 - \frac{\pi}{3} + \frac{\pi}{4}$$

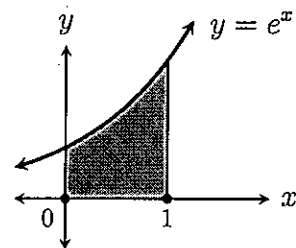
$$= \sqrt{3} - 1 - \frac{4\pi}{12} + \frac{3\pi}{12} = \boxed{\sqrt{3} - 1 - \frac{\pi}{12} \text{ square units}}$$



2. The shaded region below is rotated around the y -axis. Find the volume of the resulting solid.

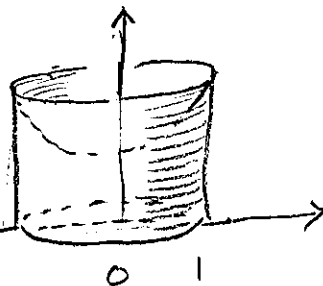
Volume by shells:

$$V = \int_0^1 2\pi x e^x dx = 2\pi \int_0^1 x e^x dx$$



$$= 2\pi \left[x e^x - e^x \right]_0^1 = 2\pi (1 \cdot e^1 - e^1 - (0e^0 - e^0))$$

$$= 2\pi (e - e + 1) = \boxed{2\pi \text{ cubic units}}$$



$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x$$

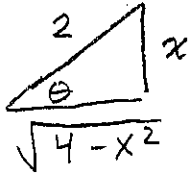
$$\begin{aligned} u &= x & dv &= e^x dx \\ du &= dx & v &= e^x \end{aligned}$$

$$3. \int \frac{dx}{x^2 \sqrt{4-x^2}} = \int \frac{2 \cos \theta d\theta}{(2 \sin \theta)^2 \sqrt{4-(2 \sin \theta)^2}} = \int \frac{2 \cos \theta d\theta}{4 \sin^2 \theta \sqrt{4-4 \sin^2 \theta}}$$

$$x = 2 \sin(\theta)$$

$$dx = 2 \cos(\theta) d\theta$$

$$\sin \theta = \frac{x}{2}$$



$$= \int \frac{2 \cos \theta d\theta}{4 \sin^2 \theta \cdot 2 \cos \theta} = \frac{1}{4} \int \frac{1}{\sin^2 \theta} d\theta$$

$$= \frac{1}{4} \int \csc^2(\theta) d\theta = -\frac{1}{4} \cot(\theta) + C$$

$$= -\frac{1}{4} \frac{\text{ADJ}}{\text{OPP}} + C = \boxed{-\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C}$$

$$4. \int \tan^5(x) \sec^4(x) dx = \int \tan^5(x) \sec^2(x) \sec^2(x) dx$$

$$= \int \tan^5(x) (1 + \tan^2(x)) \sec^2(x) dx$$

$$\begin{cases} u = \tan x \\ du = \sec^2 x dx \end{cases}$$

$$= \int u^5 (1 + u^2) du = \int u^5 + u^7 du$$

$$= \frac{u^6}{6} + \frac{u^8}{8} + C$$

$$= \boxed{\frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + C}$$

$$5. \int \frac{4x^2 + 6x + 1}{x^2 + x} dx = \int 4 + \frac{2x+1}{x^2+x} dx = \boxed{4x + \ln|x^2+x| + C}$$

$$\begin{array}{r} 4 \\ x^2 + x \overline{) 4x^2 + 6x + 1} \\ \underline{4x^2 + 4x} \\ 2x + 1 \end{array}$$

$$6. \int x^3 e^{x^2} dx = \int x^2 e^{x^2} x dx = x^2 \frac{1}{2} e^{x^2} - \int \frac{1}{2} e^{x^2} 2x dx.$$

$$\begin{aligned} u &= x^2 & dv &= e^{x^2} x dx \\ du &= 2x dx & v &= \frac{1}{2} e^{x^2} \end{aligned}$$

$$= \frac{x^2 e^{x^2}}{2} - \int e^{x^2} x dx$$

$$= \frac{x^2 e^{x^2}}{2} - \frac{1}{2} e^{x^2} + C$$

$$= \boxed{\frac{e^{x^2}}{2} (x^2 - 1) + C}$$

$$7. \int \frac{x}{x^2 - 2x + 1} dx = \int \frac{x}{(x-1)^2} dx = \int \frac{1}{x-1} + \frac{1}{(x-1)^2} dx$$

$$= \boxed{\ln|x-1| - \frac{1}{x-1} + C}$$

$$\frac{x}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$x = A(x-1) + B$$

$$\rightarrow x = Ax - A + B$$

$$\boxed{A=1}, \quad \boxed{B=1}$$

$$8. \int \frac{x-1}{x^2+3} dx = \int \frac{x}{x^2+3} - \frac{1}{x^2+3} dx$$

$$= \frac{1}{2} \int \frac{2x}{x^2+3} dx - \int \frac{1}{x^2+\sqrt{3}^2} dx$$

$$= \boxed{\frac{1}{2} \ln|x^2+3| - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$

$$9. \int_5^{\infty} \frac{4}{x^3} dx = \lim_{b \rightarrow \infty} \int_5^b \frac{4}{x^3} dx = \lim_{b \rightarrow \infty} \left[\frac{-2}{x^2} \right]_5^b$$

$$= \lim_{b \rightarrow \infty} \left(-\frac{2}{b^2} - \frac{-2}{5^2} \right) = -0 + \frac{2}{25} = \boxed{\frac{2}{25}}$$

$$10. \int_0^1 \ln(x) dx = \lim_{a \rightarrow 0^+} \int_a^1 \ln(x) dx \quad (\text{Note that } \ln(x) \text{ is not continuous on } [0, 1]!) \\ \text{Not continuous at } 0!$$

$$= \lim_{a \rightarrow 0^+} \left[x \ln(x) - x \right]_a^1$$

$$= \lim_{a \rightarrow 0^+} \left((1 \ln(1) - 1) - (a \ln a - a) \right)$$

$$= \lim_{a \rightarrow 0^+} (1 \cdot 0 - 1 - a \ln a + a)$$

$$= -1 - \lim_{a \rightarrow 0^+} a \ln a + \lim_{a \rightarrow 0^+} a$$

$$= -1 - \lim_{a \rightarrow 0^+} \frac{\ln a}{\frac{1}{a}} + 0 = -1 - \lim_{a \rightarrow 0^+} \frac{\frac{1}{a}}{-\frac{1}{a^2}}$$

form $\frac{\infty}{\infty}$

$$= -1 - \lim_{a \rightarrow 0^+} (-a)$$

$$= -1 - 0 = \boxed{-1}$$