Name: Richard

QUIZ 10 ♠

MATH 211

February 21, 2023

1. How many 10-digit binary strings are there that do not have exactly four 1's? Show and explain your work fully.

All together, there are 2^{10} 10-digit binary strings.

Also, there are $\binom{10}{4}$ binary strings with exactly four 1's. (Choose 4 of 10 spots for 1's and fill the rest with 0's.)

By the subtraction principle, the number of 10-digit binary strings that don't have exactly four 1's is

$$2^{10} - \binom{10}{4} = 1024 - \frac{10!}{6!4!} = 1024 - \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2}$$

$$= \boxed{814}$$

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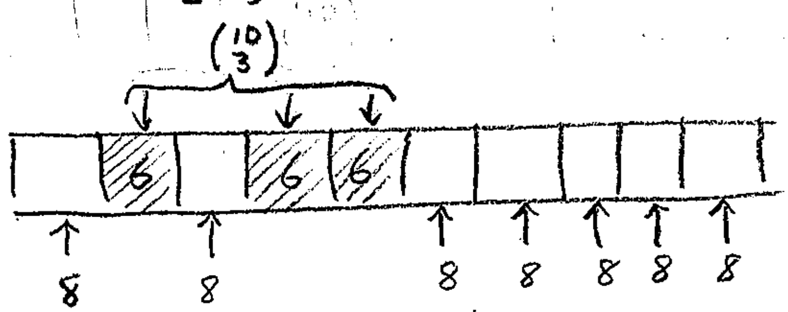
QUIZ 10 ♢

MATH 211

February 21, 2023

1. How many positive 10-digit integers contain no 0's and exactly three 6's? Show and explain your work fully.

There are $\binom{10}{3}$ ways to choose 3 positions for the 6's.



For each of the $\binom{10}{3}$ choices for the 6's, we have 8 choices (1, 2, 3, 4, 5, 7, 8, 9) for the remaining 7 spots.

Total # of lists: $\binom{10}{3} 8^7 = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} 8^7 = \boxed{251{,}658{,}240}$

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QUIZ 10 ♣

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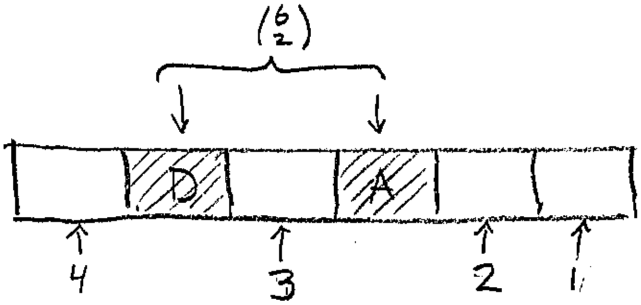
1. This problem concerns lists of length 6 made from the letters A, B, C, D, E, F, without repetition. How many such lists are there for which the D occurs before the A? Show and explain your work.

First, choose 2 of 6 spots for the D and A. There are $\binom{6}{2}$ ways to do this. For each choice, fill in the D first, followed by the A



For each of the $\binom{6}{2}$ choices, fill in the remaning spots with the letters B, C, E, F. (see above).

<u>Total # of lists:</u> $\binom{6}{2} 4\cdot3\cdot2\cdot1 = \frac{6!}{4!\,2!} 4\cdot3\cdot2\cdot1 = \frac{6!}{2} = \boxed{360}$

Name: Richard

QUIZ 10 ♡

MATH 211

February 21, 2023

1. How many 10-digit binary strings are there that have exactly four 1's or exactly five 1's? Show and explain your work fully.

There are $\binom{10}{4}$ 10-digit binary strings with exactly four 1's. (Choose 4 out of 10 spots for the 1's and fill the remaining spots with 0's)

There are $\binom{10}{5}$ 10-digit binary strings with exactly five 1's. (Choose 5 out of 10 spots for the 1's, etc.)

By the addition principle, the total number of such strings is

$$\binom{10}{4} + \binom{10}{5} = \frac{10!}{6!\,4!} + \frac{10!}{5!\,5!} = \frac{10\cdot9\cdot8\cdot7}{4\cdot3\cdot2} + \frac{10\cdot9\cdot8\cdot7\cdot6}{5\cdot4\cdot3\cdot2} = \boxed{462}$$