MATH 211 Test #1 ♠ March 14, 2023

Name: Richard R. Hammack Score:______

Directions Except in a problem designated **short answer**, you must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified, as in, for example, $7^{15} - 7!$. All you will need is something to write with. Scratch paper will be provided.

1. (9 points) **Short answer.**

   (a) Write $\{\ldots -5, -2, 1, 4, 7, 10, 13, 16\ \ldots\}$ in set-builder notation.

   $\boxed{\{1+3n : n \in \mathbb{Z}\}}$

   (b) Write the set $\left\{\frac{n}{n+1} : 1 \le n \le 3\right\}$ by listing its elements between braces.

   $\boxed{\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}}$

   (c) $\mathscr{P}(\{1,2\}) = \boxed{\{\phi, \{1\}, \{2\}, \{1,2\}\}}$

2. (12 points) **Short answer.** Suppose $A = \{1,2\}$ and $B = \{2,4\}$.

   (a) $A \times A = \boxed{\{(1,1), (1,2), (2,1), (2,2)\}}$

   (b) $A \times B = \boxed{\{(1,2), (1,4), (2,2), (2,4)\}}$

   (c) $(A \times A) - (A \times B) = \boxed{\{(1,1), (2,1)\}}$

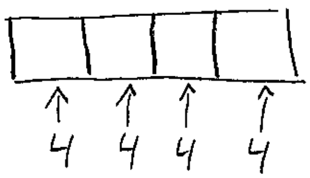
   (d) $(A \times A) \cap (A \times B) = \boxed{\{(1,2), (2,2)\}}$

3. (9 points) Write a truth table for the expression $\neg P \vee (Q \Rightarrow R)$.

P	Q	R	$\neg P$	$Q \Rightarrow R$	$\neg P \vee (Q \Rightarrow R)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

4. (20 points) Consider **length-4** lists made from the symbols A, B, C, D, with **repetition allowed.**

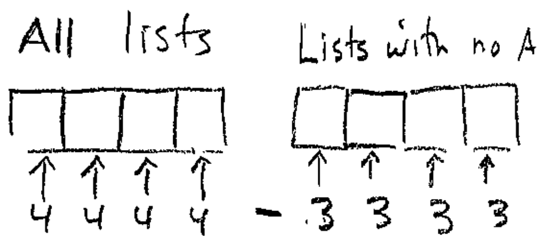
(a) How many such lists are there?



Answer: $4^4 = \boxed{256}$

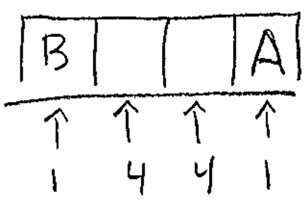
(b) How many such lists are there that have at least one A?

<u>Subtraction principle:</u>



Answer: $4^4 - 3^4 = \boxed{175}$

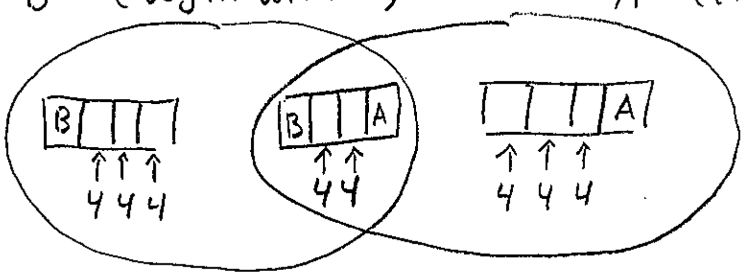
(c) How many such lists are there that begin with B **and** end with A?



Answer: $1 \cdot 4 \cdot 4 \cdot 1 = \boxed{16}$

(d) How many such lists are there that begin with B **or** end with A?

B (begin with B) A (end in A)



Inclusion - Exclusion

<u>Answer</u>: $|A \cup B| = |A| + |B| - |A \cap B|$

$= 4^3 + 4^3 - 4^2 = 64 + 64 - 16 = \boxed{112}$

5. (8 points) Consider the length-7 lists made from the symbols A, B, C, D, with repetition allowed. How many such lists are in alphabetical order? (For example, $AABBBCD$ or $BBBCCCD$, but **not** $BBAADAD$, etc.)

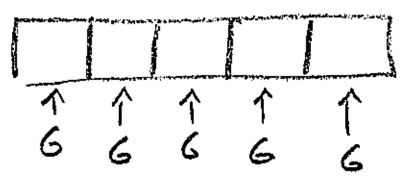
Think of such a list with groups of letters separated by a bar, such as AA|BB|CC|D or A|BBB|CCC| , etc.

This can be modeled as a star and bar list **|**|**|* with 7 stars and 3 bars. Such a list has length 10. Make it by selecting 3 of 10 positions for bars and filling remaining positions with stars. The number of such lists is $\binom{10}{3} = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8}{3\cdot 2}$

$= 10\cdot 3\cdot 4 = \boxed{120}$

6. (8 points) Imagine tossing a 6-sided dice five times. A typical outcome can be described as a length-5 list such as 43461, meaning you rolled a 4 first, then 3, then 4, then 6, then 1.
How many outcomes are there in which not all tosses are odd? (e.g., 12134 or 22462, but not 31551)

All possible outcomes



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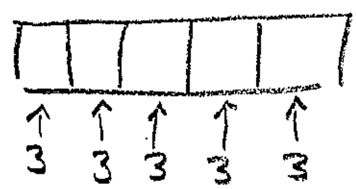
All tosses are odd



By the subtraction principle, the answer is $6^5 - 3^5 = 7776 - 243$

$= \boxed{7533}$

7. (10 points)

(a) How many subsets $X \subseteq \{1,2,3,4,5,6,7,8,9\}$ are there for which $|X| = 5$?

$$\binom{9}{5} = \frac{9!}{4!\,5!} = \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2} = 3\cdot 7\cdot 6 = \boxed{126}$$

(b) Write Pascal's triangle to the 5th row and use it to expand $(x+y)^5$.

1

1 1

1 2 1

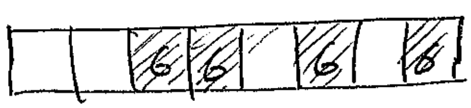
1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$\boxed{(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5}$$

8. (8 points) How many 8-digit positive integers have no 0's and exactly four 6's?



Start by choosing 4 out of 8 places for the 6's. There are $\binom{8}{4}$ ways to make this selection. Once that is done, the remaining 4 positions can be filled in 8 ways each (no 0's & no 6's)

Answer: $\binom{8}{4}8^4 = \frac{8!}{4!\,4!}8^4 = \frac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2}8^4 = 7\cdot 2\cdot 5\cdot 8^4$

$= 70\cdot 8^4 = \boxed{286{,}720}$

9. (8 points) You deal six cards from a 52-card deck and line them up in a row. How many possible 6-card lineups are there where no two cards of the same color are next to one another?
(A deck has 26 red cards and 26 black cards.)

Two types of lineups:

R	B	R	B	R	B
26 ·	26 ·	25 ·	25	24	24

B	R	B	R	B	R
↑	↑	↑	↑	↑	↑
26	26	25	25	24	24

By the addition principle, the total # of lineups is $2 \cdot 26^2 \cdot 25^2 \cdot 24^2 = \boxed{486,720,000}$

10. (8 points) A bag contains 20 red balls, 20 blue balls, 20 green balls and 20 white balls. You reach in and take 6 balls. How many different outcomes are possible?

Your outcome: Reds Blues Greens Whites
*** | **** | *** | **

list, 6 stars, 3 bars, length $6+3=9$
Make such a list by choosing 3 of 9 positions for bars, and filling rest with stars

Ans

$$\binom{9}{3} = \frac{9!}{6!\,3!} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 3 \cdot 4 \cdot 7 = \boxed{84}$$