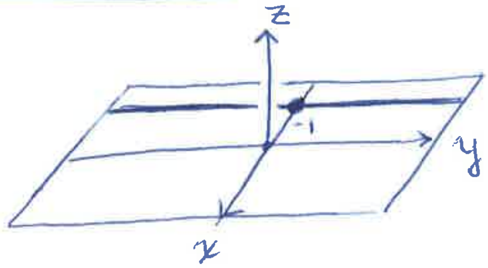
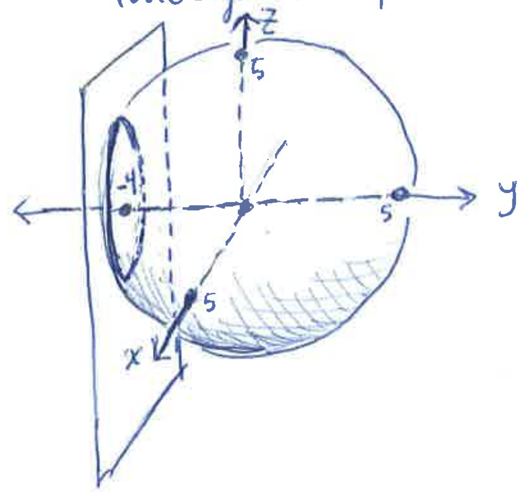
MATH 307 Homework #1

Section 12.1

(2) $x=-1$, $z=0$.



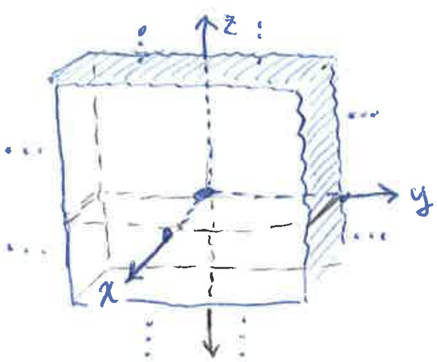
This is the line on the x-y plane parallel to the y-axis and passing through the point $(-1,0,0)$.



(10) $\underbrace{x^2+y^2+z^2=25}_{\text{sphere of radius 5 centered at origin.}}$, $\underbrace{y=-4}_{\text{plane parallel to xz-plane at } y=-4}$

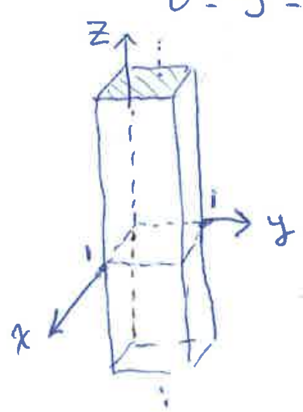
The set of points (x,y,z) that satisfy both of these equations are the points on the intersection of the sphere and the plane, illustrated above. Since $y=-4$, they satisfy $x^2+(-4)^2+z^2=25$ or $x^2+y^2=3^2$. This is a circle of radius 3 on the plane $y=-4$ centered at the point $(0,-4,0)$

(18) (a) $0 \le x \le 1$



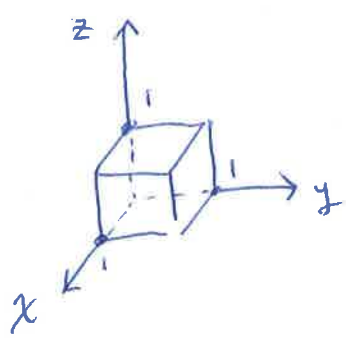
This is an infinite "slab", one unit thick in the x direction

(b) $0 \le x \le 1$
$0 \le y \le 1$



This is an infinitely tall box, as illustrated

(c) $0 \le x \le 1$
$0 \le y \le 1$
$0 \le z \le 1$



This is a solid box of dimensions $1 \times 1 \times 1$, as illustrated

30. Describe with equations: Circle of radius 1 centered at $(-3, 4, 1)$ and lying in a plane parallel to:

(a) xy-plane: $\boxed{\begin{cases} (x+3)^2 + (y-4)^2 = 1 \\ z = 1 \end{cases}}$

(b) yz-plane $\boxed{\begin{cases} (y-4)^2 + (z-1)^2 = 1 \\ x = -3 \end{cases}}$

(c) xz-plane $\boxed{\begin{cases} (x+3)^2 + (z-1)^2 = 1 \\ y = 4 \end{cases}}$

42. Find the distance between $P_1(-1, 1, 5)$ & $P_2(2, 5, 0)$

Answer: distance $= \sqrt{(2-(-1))^2 + (5-1)^2 + (0-5)^2} = \sqrt{3^2+4^2+5^2}$

$= \sqrt{50} = \sqrt{2 \cdot 25} = \boxed{5\sqrt{2} \text{ units}}$

48. Sphere: $(x-1)^2 + (y+\frac{1}{2})^2 + (z+3)^2 = 25$

$\rightsquigarrow (x-1)^2 + (y-(\frac{-1}{2}))^2 + (z-(-3))^2 = 5^2$

$\boxed{\text{Center: } (1, -\tfrac{1}{2}, -3), \quad \text{radius: } 5}$

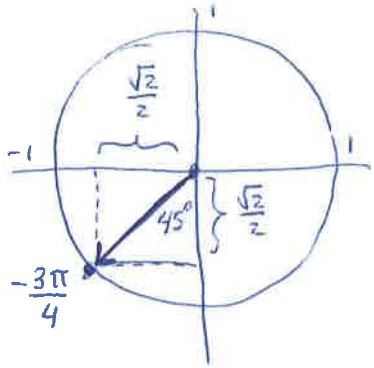
Section 12.2

6. $-2\vec{u} + 5\vec{v} = -2\langle 3, -2\rangle + 5\langle -2, 5\rangle = \langle -6, 4\rangle + \langle -10, 25\rangle = \boxed{\langle -16, 29\rangle}$

Length: $\sqrt{(-16)^2 + (29)^2} = \boxed{\sqrt{1097}}$

10. $R = (2, -1)$ $S = (-4, 3)$. Midpoint of RS is $\left(\frac{2-4}{2}, \frac{-1+3}{2}\right) = (-1, 1)$

Let $P = (-1, 1)$. Problem asks for $\overrightarrow{OP} = \langle -1-0, 1-0\rangle = \boxed{\langle -1, 1\rangle}$

14 

From the diagram, the unit vector at $\theta = -\frac{3\pi}{4}$ is $\boxed{\vec{u} = \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle}$

18 Let $P_1 = (1,2,0)$, $P_2 = (-3,0,5)$

Then $\overrightarrow{P_1P_2} = \langle -3-1, 0-2, 5-0 \rangle = \langle -4, -2, 5 \rangle = \boxed{-4\vec{i} - 2\vec{j} + 5\vec{k}}$

22 $-2\vec{u} + 3\vec{v} = -2\langle -1, 0, 2 \rangle + 3\langle 1, 1, 1 \rangle = \langle 2, 0, -4 \rangle + \langle 3, 3, 3 \rangle$

$= \langle 5, 3, -1 \rangle = \boxed{5\vec{i} + 3\vec{j} - \vec{k}}$

26 $9\vec{i} - 2\vec{j} + 6\vec{k} = \langle 9, -2, 6 \rangle$ has length $\sqrt{9^2 + (-2)^2 + 6^2}$

$= \sqrt{81 + 4 + 36} = \sqrt{121} = 11$. Thus a unit vector in its direction is $\left\langle \frac{9}{11}, \frac{-2}{11}, \frac{6}{11} \right\rangle$ Therefore:

$$9\vec{i} - 2\vec{j} + 6\vec{k} = \underset{\text{length}}{11} \cdot \underbrace{\left\langle \frac{9}{11}, \frac{-2}{11}, \frac{6}{11} \right\rangle}_{\text{direction}}$$

36 $P_1(1, 4, 5)$ $P_2(4, -2, 7)$

(a) Midpoint: $\left(\frac{1+4}{2}, \frac{4-2}{2}, \frac{5+7}{2} \right) = \boxed{\left(\frac{5}{2}, 1, 6 \right)}$

(b) $\overrightarrow{P_1P_2} = \langle 4-1, -2-4, 7-5 \rangle = \langle 3, -6, 2 \rangle$

$|\overrightarrow{P_1P_2}| = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

$\boxed{\overrightarrow{P_1P_2} = 7 \left\langle \frac{3}{7}, \frac{1}{7}, \frac{6}{7} \right\rangle}$ (gives $\overrightarrow{P_1P_2}$ as a product of its length and direction)

(40) If $\overrightarrow{AB} = -7\vec{i} + 3\vec{j} + 8\vec{k}$ and $A = (-2, -3, 6)$, then find B.

Solution: Let $B = \langle x, y, z \rangle$. Then

$$\overrightarrow{AB} = \langle -7, 3, 8 \rangle = \langle x-(-2), y-(-3), z-6 \rangle$$
$$= \langle x+2, y+3, z-6 \rangle$$

Then: $-7 = x+2 \rightsquigarrow x = -9$

$3 = y+3 \rightsquigarrow y = 0$

$8 = z-6 \rightsquigarrow z = 14$

Therefore: $\boxed{B = \langle -9, 0, 14 \rangle}$