MATH 307 Homework # 2

Section 12.3

(4) $\vec{v} = \langle 2, 10, -11\rangle \quad \vec{u} = \langle 2, 2, 1\rangle$

(a) $\vec{u}\cdot\vec{v} = 2\cdot 2 + 10\cdot 2 - 11\cdot 1 = 4+20-11 = \boxed{13}$

$|\vec{v}| = \sqrt{2^2+10^2+(-11)^2} = \sqrt{4+100+121} = \sqrt{225} = \boxed{15}$

$|\vec{u}| = \sqrt{2^2+2^2+1^2} = \sqrt{9} = \boxed{3}$

(b) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{13}{15\cdot 3} = \boxed{\dfrac{13}{45}}$

(c) $\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|} = \boxed{\dfrac{13}{15}}$

(d) $\text{proj}_{\vec{v}}\,\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v} = \dfrac{13}{\quad}\,\vec{v} = \boxed{\dfrac{13}{225}\langle 2, 10, -11\rangle}$

(8) $\vec{v} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right\rangle \quad \vec{u} = \left\langle \frac{1}{\sqrt{2}}\ -\frac{1}{\sqrt{3}}\right\rangle$

(a) $\vec{u}\cdot\vec{v} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}} = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}}$

$|\vec{u}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{3}} = \sqrt{\frac{5}{6}}$

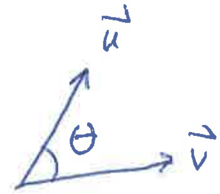
$|\vec{v}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{3}} = \sqrt{\frac{5}{6}}$

(b) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{1/6}{\sqrt{\frac{5}{6}}\sqrt{5/6}} = \dfrac{1/6}{5/6} = \boxed{\dfrac{1}{5}}$

(c) $\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|} = \dfrac{1/6}{\sqrt{5/6}} = \dfrac{1}{6}\dfrac{\sqrt{6}}{\sqrt{5}} = \dfrac{1}{\sqrt{6}\sqrt{6}}\dfrac{\sqrt{6}}{\sqrt{5}} = \dfrac{1}{\sqrt{6}\sqrt{5}} = \boxed{\dfrac{1}{\sqrt{30}}}$

(d) $\text{Proj}_{\vec{v}}\,\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v} = \dfrac{1/6}{5/6}\,\vec{v} = \dfrac{1}{5}\left\langle \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{3}}\right\rangle$

Section 12.3 (continued)



(10) $\vec{u} = 2\vec{i} - 2\vec{j} + \vec{k} = \langle 2, -2, 1 \rangle$

$\vec{v} = 3\vec{i} + 4\vec{k} = \langle 3, 0, 4 \rangle$

$$\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}\right) = \cos^{-1}\left(\frac{10}{3\cdot 5}\right) = \cos^{-1}\left(\frac{10}{15}\right)$$

$$= \cos^{-1}\left(\frac{2}{3}\right) \approx \boxed{0.84 \text{ radians}}$$

Section 12.4

(8) $\vec{u} = \langle \frac{3}{2}, -\frac{1}{2}, 1 \rangle \qquad \vec{v} = \langle 1, 1, 2 \rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{3}{2} & -\frac{1}{2} & 1 \\ 1 & 1 & 2 \end{vmatrix} = \boxed{\langle -2, -2, 2 \rangle}$$

$$\vec{v}\times\vec{u} = -\vec{u}\times\vec{v} = -\langle -2, -2, 2 \rangle = \boxed{\langle 2, 2, -2 \rangle}$$

The length of each of these vectors is

$$|\vec{u}\times\vec{v}| = |\vec{v}\times\vec{u}| = \sqrt{4+4+4} = \sqrt{12} = \boxed{2\sqrt{3}}$$

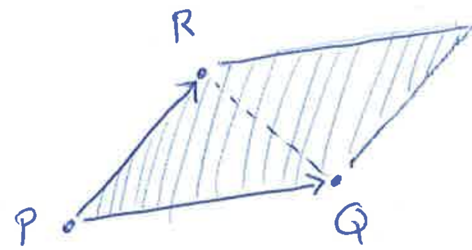
Direction of $\vec{u}\times\vec{v}$ is $\frac{1}{2\sqrt{3}}\langle -2, -2, 2 \rangle = \langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

Direction of $\vec{v}\times\vec{u}$ is $\frac{1}{2\sqrt{3}}\langle 2, 2, -2 \rangle = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle$

Section 12.4 (Continued)

(16) (a) Find the area of the triangle with corners
P(1,1,1) Q(2,1,3) R(3,-1,1)

Note Area of triangle is half the area of the parallelogram formed by $\vec{PR}$ and $\vec{PQ}$.



Thus Area $= \frac{1}{2} |\vec{PQ} \times \vec{PR}|$.

Need to find: $\vec{PQ} = \langle 2-1, 1-1, 3-1 \rangle = \langle 1, 0, 2 \rangle$
$\vec{PR} = \langle 3-1, -1-1, 1-1 \rangle = \langle 2, -2, 0 \rangle$

Then $\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix} = \langle 4, 4, -2 \rangle$

Then Area $= \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} \sqrt{4^2 + 4^2 + (-2)^2}$

$= \frac{1}{2} \sqrt{36} = \frac{1}{2} \cdot 6 = \boxed{3 \text{ square units}}$

(b) Find a unit vector perpendicular to the plane containing this triangle

Answer $\dfrac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} = \dfrac{\langle 4, 4, -2 \rangle}{6} = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right\rangle$

Section 12.4 (Continued)

(24) $\vec{u} = \langle 1, 2, -1 \rangle$

$\vec{v} = \langle -1, 1, 1 \rangle$

$\vec{w} = \langle 1, 0, 1 \rangle$

$\vec{r} = \left\langle -\frac{\pi}{2}, -\pi, \frac{\pi}{2} \right\rangle$

(a) $\vec{u} \cdot \vec{v} = 0$ so $\vec{u} \perp \vec{v}$

$\vec{u} \cdot \vec{w} = 0$ so $\vec{u} \perp \vec{w}$

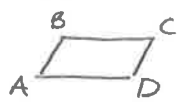
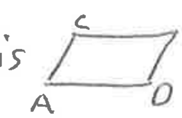
$\vec{v} \cdot \vec{w} = 0$ so $\vec{v} \perp \vec{w}$

$\vec{v} \cdot \vec{r} = 0$ so $\vec{v} \perp \vec{r}$

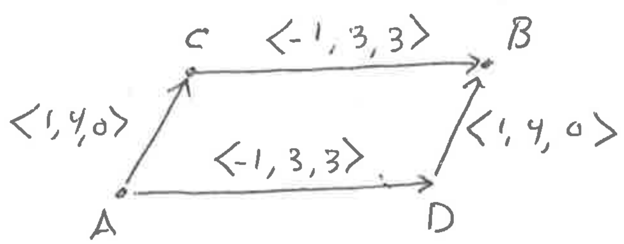
$\vec{w} \cdot \vec{r} = 0$ so $\vec{w} \perp \vec{r}$

(b) $\vec{u}$ is a scalar multiple of $\vec{r}$, so these vectors are parallel.

(40) Find the area of the parallelogram whose vertices are $A(1,0,-1)$, $B(1,7,2)$, $C(2,4,-1)$, $D(0,3,2)$

First we need to find the relative positions of the corners, i.e., is it like this  or this  etc?

A little trial and error reveals the parallelogram:



Not opposite sides are parallel

Then Area $= |AC \times AD| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 0 \\ -1 & 3 & 3 \end{vmatrix} \right\| = \left| \langle 12, -3, 7 \rangle \right|$

$= \sqrt{12^2 + 3^2 + 7^2} = \sqrt{144 + 9 + 49} = \sqrt{202}$ sq. units.