MATH 490 Homework #3

Section 12.5

(2) Find the parametric equations for the line through points $P(1,2,-1)$ and $Q(-1,0,1)$.

This line is parallel to the vector $\overrightarrow{QP} = \langle 1-(-1), 2-0, -1-1 \rangle$ $= \langle 2, 2, -2 \rangle$. Also it passes through $Q = \langle -1, 0, 1 \rangle$. Thus its vector equation is $\vec{r}(t) = \langle -1, 0, 1 \rangle + t\langle 2, 2, -2 \rangle = \langle -1+2t, 2t, 1-2t \rangle$

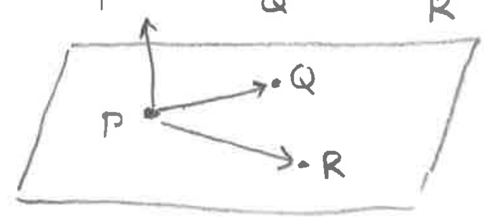
Consequently its parametric form is $\boxed{\begin{cases} x = -1+2t \\ y = 2t \\ z = 1-2t \end{cases} \quad -\infty < t < \infty.}$

(24) Find the equation of the plane through $\underbrace{(2,4,5)}_{P}$ $\underbrace{(1,5,7)}_{Q}$ $\underbrace{(-1,6,8)}_{R}$

Following vectors are on this plane:

$\overrightarrow{PQ} = \langle 1-2, 5-4, 7-5 \rangle = \langle -1, 1, 2 \rangle$

$\overrightarrow{PR} = \langle -1-2, 6-4, 8-5 \rangle = \langle -3, 2, 3 \rangle$



Normal is $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix} = \langle -1, -3, 1 \rangle$

Equation of plane: $-x - 3y + z = 2(-1) + 4(-3) + 5 \cdot 1$

$-x - 3y + z = -9$

$\boxed{x + 3y - z = 9}$

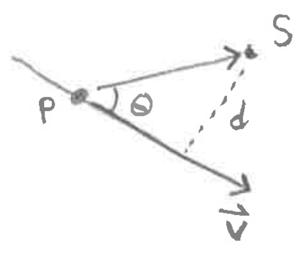
(36) Find the distance from $S(2, 1, -1)$ to the line $\begin{cases} x = 2t \\ y = 1+2t \\ z = 2t \end{cases}$

For $t=0$, we see $P(0,1,0)$ is on the line.

Also the line has direction $\langle 2, 2, 2 \rangle$, but we can use $\vec{v} = \langle 1, 1, 1 \rangle$

Now $\overrightarrow{PS} = \langle 2, 0, -1 \rangle$

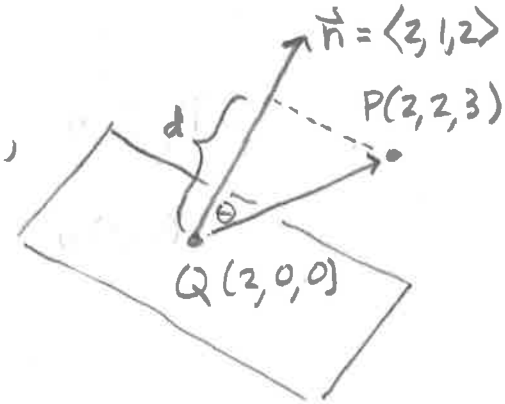
$d = |\overrightarrow{PS}| \sin\theta = \dfrac{|\overrightarrow{PS}||\vec{v}| \sin\theta}{|\vec{v}|} = \dfrac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$



$= \dfrac{|\langle -1, 3, -2 \rangle|}{\sqrt{3}} = \dfrac{\sqrt{14}}{\sqrt{3}} = \boxed{\sqrt{\frac{14}{3}} \text{ units}}$

(42) Find the distance from point $P(2,2,3)$ to the plane $2x + y + 2z = 4$.

A point on the plane is $Q(2,0,0)$, and the normal to the plane is $\vec{n} = \langle 2,1,2 \rangle$. Also $\overrightarrow{QP} = \langle 0,2,3 \rangle$



Then distance $= d = |\overrightarrow{QP}| \cos\theta$

$$= \frac{|\overrightarrow{QP}||\vec{n}|\cos\theta}{|\vec{n}|} = \frac{\overrightarrow{QP}\cdot\vec{n}}{|\vec{n}|} = \frac{\langle 0,2,3\rangle \cdot \langle 2,1,2\rangle}{\sqrt{2^2+1^2+2^2}}$$

$$= \frac{8}{\sqrt{9}} = \boxed{\frac{8}{3} \text{ units}}$$

(54) Find the point where the line $\begin{cases} x = 2 \\ y = 3+2t \\ z = -2-2t \end{cases}$ meets the plane $6x + 3y - 4z = -12$.

Solution: This point is given by the value of t for which

$$6\cdot 2 + 3(3+2t) - 4(-2-2t) = -12$$
$$12 + 9 + 6t + 8 + 8t = -12$$
$$14t = -41$$
$$\boxed{t = -\frac{41}{14}}$$

Thus the point of intersection is

$x = 2$

$y = 3 + 2t = 3 + 2\left(-\frac{41}{14}\right) = 3 - \frac{41}{7} = -\frac{20}{7}$

$z = -2 - 2t = -2 - 2\left(-\frac{41}{14}\right) = -2 + \frac{41}{7} = \frac{27}{7}$

ANSWER Point is $\boxed{\left(2, -\frac{20}{7}, \frac{27}{7}\right)}$