MATH 307 Homework #4

Section 13.1

Note $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

(10) $\vec{r}(t) = \left\langle 1+t, \frac{t^2}{\sqrt{2}}, \frac{t^3}{3} \right\rangle$

Velocity vector: $\vec{v}(t) = \vec{r}\,'(t) = \langle 1, \sqrt{2}\,t, t^2 \rangle$

Acceleration vector: $\vec{a}(t) = \vec{v}\,'(t) = \langle 0, \sqrt{2}, 2t \rangle$

At $t=1$, velocity is $v(1) = \langle 1, \sqrt{2}, 1 \rangle$

$$= |\langle 1, \sqrt{2}, 1\rangle| \, \frac{\langle 1, \sqrt{2}, 1 \rangle}{|\langle 1, \sqrt{2}, 1\rangle|}$$

$$= \frac{2}{2}\langle 1, \sqrt{2}, 1 \rangle$$

$$= \underbrace{2}_{\text{speed}} \underbrace{\left\langle \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2} \right\rangle}_{\text{direction}}$$

From calculation on right:
Speed at $t=1$ is $2 \frac{\text{units}}{\text{sec}}$
Direction at $t=1$ is $\left\langle \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2} \right\rangle$

(18) $\vec{r}(t) = \left\langle \frac{4}{9}(1+t)^{3/2}, \frac{4}{9}(1-t)^{3/2}, \frac{1}{3}t \right\rangle$

Velocity: $\vec{v}(t) = \vec{r}\,'(t) = \left\langle \frac{2}{3}(1+t)^{\frac{1}{2}}, -\frac{2}{3}(1+t)^{\frac{1}{2}}, \frac{1}{3} \right\rangle$

$$= \boxed{\left\langle \frac{2}{3}\sqrt{1+t}, -\frac{2}{3}\sqrt{1-t}, \frac{1}{3} \right\rangle}$$

acceleration $\vec{a}(t) = \vec{v}\,'(t) = \boxed{\left\langle \frac{1}{3\sqrt{1+t}}, \frac{1}{3\sqrt{1+t}}, 0 \right\rangle}$

Now, $\vec{v}(0) = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$

$\vec{a}(0) = \left\langle \frac{1}{3}, \frac{1}{3}, 0 \right\rangle$

Angle between these vectors is $\cos^{-1}\left(\frac{\vec{v}(0) \cdot \vec{a}(0)}{|\vec{v}(0)|\,|\vec{a}(0)|} \right)$

$= \cos^{-1}(0) = \frac{\pi}{2}$, or $\boxed{90^\circ}$

MATH 307

Section 13.2

(4) $\displaystyle\int_0^{\pi/3} \left\langle \sec t \tan t,\ \tan t,\ 2 \sin t \cos t \right\rangle dt$

$= \left[\left\langle \sec t,\ -\ln|\cos t|,\ \sin^2 t \right\rangle \right]_0^{\pi/3}$

$= \left\langle \sec \frac{\pi}{3},\ -\ln\left|\cos \frac{\pi}{3}\right|,\ \sin^2 \frac{\pi}{3} \right\rangle - \left\langle \sec 0,\ -\ln|\cos 0|,\ \sin^2 0 \right\rangle$

$= \left\langle 2,\ -\ln \frac{1}{2},\ \left(\frac{\sqrt{3}}{2}\right)^2 \right\rangle - \langle 1, 0, 0 \rangle = \boxed{\left\langle 1,\ \ln 2,\ \frac{3}{4} \right\rangle}$

(14) Solve $\vec{r}\,'(t) = \langle t^3 + 4t,\ t,\ 2t^2 \rangle$ subject to $\vec{r}(0) = \langle 1, 1, 0 \rangle$.

$\vec{r}(t) = \displaystyle\int \langle t^3 + 4t,\ t,\ 2t^2 \rangle \, dt$

$= \left\langle \frac{t^4}{4} + 2t^2,\ \frac{t^2}{2},\ \frac{2t^3}{3} \right\rangle + \vec{C}$.

$= \langle 0, 0, 0 \rangle + \vec{C}$

Note: $\langle 1, 1, 0 \rangle = \vec{r}(0) = \langle 0, 0, 0 \rangle + \vec{C}$

Thus $\vec{C} = \langle 1, 1, 0 \rangle$, so

$\vec{r}(t) = \left\langle \frac{t^4}{4} + 2t^2 + 1,\ \frac{t^2}{2} + 1,\ \frac{2t^3}{3} \right\rangle$