Section 13.3

(8) $\vec{r}(t) = \langle t\sin t + \cos t,\ t\cos t - \sin t,\ 0 \rangle$

$\vec{r}\,'(t) = \langle \sin t + t\cos t - \sin t,\ \cos t - t\sin t - \cos t,\ 0 \rangle$

$= \langle t\cos t,\ -t\sin t,\ 0 \rangle$

$|\vec{r}\,'(t)| = \sqrt{(t\cos t)^2 + (-t\sin t)^2 + 0^2} = \sqrt{t^2\cos^2 t + t^2\sin^2 t}$

$= \sqrt{t^2(\cos^2 t + \sin^2 t)} = \sqrt{t^2} = t$. Assume t positive

Therefore unit tangent vector is $\vec{T}(t) = \dfrac{r'(t)}{|r'(t)|} =$

$\frac{1}{t}\langle t\cos t,\ -t\sin t,\ 0 \rangle = \boxed{\langle \cos t,\ -\sin t,\ 0 \rangle}$

Arc length is

$$\int_{\sqrt{2}}^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_{\sqrt{2}}^{2} \sqrt{(t\cos t)^2 + (-t\sin t)^2 + 0^2}\, dt$$

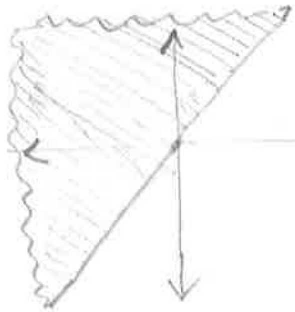
$$= \int_{\sqrt{2}}^{2} \sqrt{t^2\cos^2 t + t^2\sin^2 t}\, dt$$

$$= \int_{\sqrt{2}}^{2} \sqrt{t^2(\cos^2 t + \sin^2 t)}\, dt = \int_{\sqrt{2}}^{2} \sqrt{t^2}\, dt$$

$$= \int_{\sqrt{2}}^{2} t\, dt = \left[\frac{t^2}{2}\right]_{\sqrt{2}}^{2} = \frac{2^2}{2} - \frac{\sqrt{2}^2}{2} = 2 - 1 = \boxed{1 \text{ unit}}$$

4.1

(18) $f(x, y) = \sqrt{y - x}$



(a) Domain Must have $y - x \geq 0$
Thus domain is $\{(x,y) \mid x, y \in \mathbb{R},\ y \geq x\}$

(b) Range For any non-negative number b, observe that $f(0, b^2) = \sqrt{b^2 - 0} = b$. Thus the range contains all the non-negative reals. On the other hand, $f(x,y) = \sqrt{y-x}$ can't be negative, so the range is $[0, \infty)$

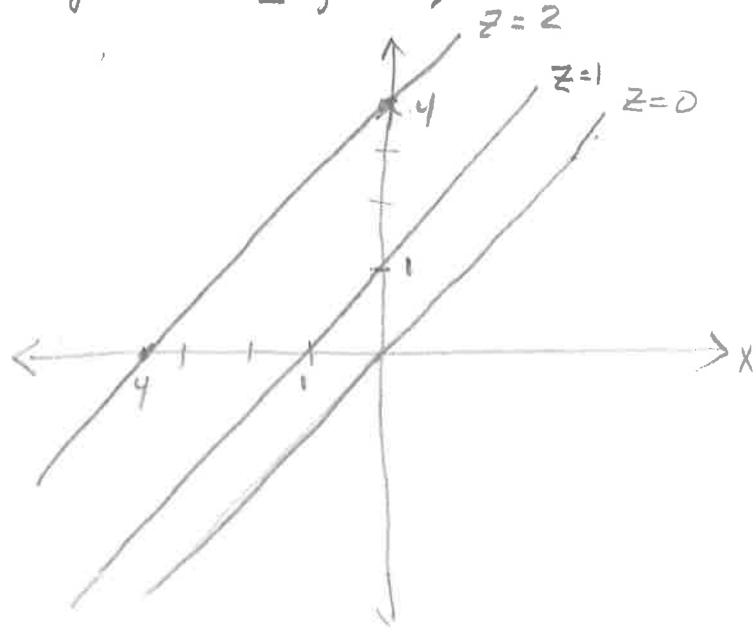
(c) Consider the level curve $z = k$.

Then $k = f(x,y) = \sqrt{y-x}$

$$k^2 = \left(\sqrt{y-x}\right)^2$$

$$k^2 = y + x$$

$$y = k^2 + x$$



Thus the level curve for $z = k$ is a straight line, y-intercept k^2 and slope 1

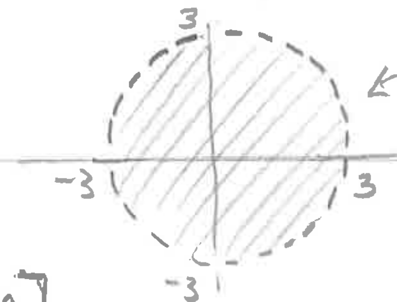
(d) The boundary is the line $y = x$

(e) Domain is a closed region. It contains its boundary points, namely all points on the line $y = x$, because for these $f(x,y) = \sqrt{y-x} = \sqrt{0} = 0$ is defined.

(f) The domain is unbounded.

4.1 (30) $f(x,y) = \ln(9 - x^2 - y^2)$

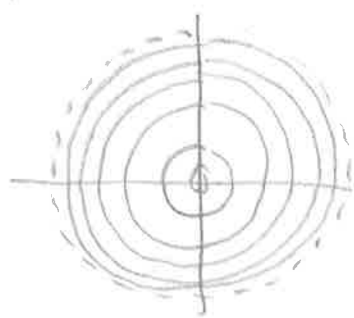
(a) The domain consists only of those points (x,y) for which $9 - x^2 - y^2 > 0$ because only a positive number can be plugged into the ln. Thus we require $x^2 + y^2 < 3^2$ and these are the points inside a circle of radius 3 centered at $(0,0)$



Domain is $\{(x,y) \mid x^2+y^2 < 3^2\}$

(b) Range is $(-\infty, \ln 9]$

(c) Level curve for $z = k$ is



$k = \ln(9 - x^2 - y^2)$

$e^k = 9 - x^2 - y^2$

$x^2 + y^2 = 9 - e^k$

Thus the level curve for $z = k$ is the circle of radius $\sqrt{9 - e^k}$ centered at the origin.

(d) The boundary of the domain is the circle of radius 3.

(e) The domain is open

(f) The domain is bounded