Math 307

Section 14.2

(10) $\displaystyle\lim_{(x,y)\to(\frac{1}{27},\pi^3)} \cos\sqrt[3]{xy} = \cos\sqrt[3]{\tfrac{1}{27}\pi^3} = \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$

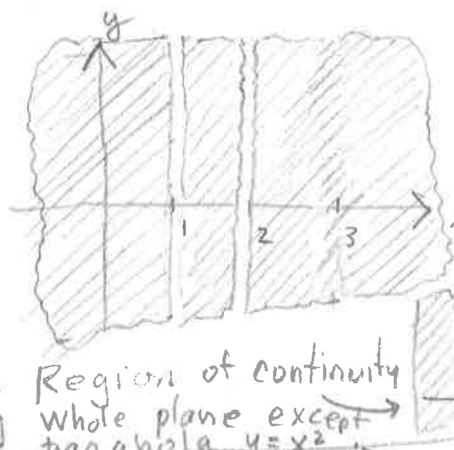
(18) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2} = \lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2}\,\frac{\sqrt{x+y}+2}{\sqrt{x+y}+2}$

$\displaystyle= \lim_{(x,y)\to(2,2)} \frac{(x+y-4)(\sqrt{x+y}+2)}{x+y-4} = \lim_{(x,y)\to(2,2)}\left(\sqrt{x+y}+2\right) = \sqrt{2+2}+2 = \boxed{4}$

(16) $\displaystyle\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y-xy+4x^2-4x} = \lim_{(x,y)\to(2,-4)} \frac{y+4}{(y+4)(x^2-x)}$

$\displaystyle= \lim_{(x,y)\to(2,-4)} \frac{1}{x^2-x} = \frac{1}{2^2-2} = \boxed{\frac{1}{2}}$

(34) (a) $\displaystyle g(x,y) = \frac{x^2+y^2}{x^2-3x+2} = \frac{x^2+y^2}{(x-1)(x-2)}$ This is a quotient of continuous functions, so it is continuous at all (x,y) except those for which the denominator $(x-1)(x-2)$ is 0. Thus it is continuous on all of the xy-plane <u>except</u> on the lines $x=1$ and $x=2$



(b) $\displaystyle g(x,y) = \frac{1}{x^2-y}$ Region of continuity Whole plane except parabola $y=x^2$

(46) $\displaystyle g(x,y) = \frac{x^2-y}{x-y}$

If $(x,y)\to(0,0)$ along the y-axis (where $x=0$) we get

$\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} = \lim_{(x,y)\to(0,0)} \frac{-y}{-y} = \boxed{1}$

If $(x,y)\to(0,0)$ along the x-axis (where $y=0$) we get

$\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} = \lim_{(x,y)\to(0,0)} \frac{x^2}{x} = \lim_{(x,y)\to(0,0)} x = \boxed{0}$

Conclusion: LIMIT D.N.E.

Section 14.3

(10) $f(x,y) = \dfrac{x}{x^2+y^2}$

$$\frac{\partial f}{\partial x} = \frac{(1)(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \boxed{\frac{y^2 - x^2}{(x^2+y^2)^2}}$$

$$\frac{\partial f}{\partial y} = x\left(-(x^2+y^2)^{-2}\, 2y\right) = \boxed{-\frac{2xy}{(x^2+y^2)^2}}$$

(18) $f(x,y) = \cos^2(3x - y^2)$

$$\frac{\partial f}{\partial x} = 2\cos(3x-y^2)\left(-\sin(3x-y^2)\cdot 3\right) = \boxed{-6\cos(3x-y^2)\sin(3x-y^2)}$$

$$\frac{\partial f}{\partial y} = 2\cos(3x-y^2)\left(-\sin(3x-y^2)(-2y)\right) = \boxed{4y\cos(3x-y^2)\sin(3x-y^2)}$$

(28) $f(x,y,z) = \sec^{-1}(x+yz)$

$$\boxed{\frac{\partial f}{\partial x} = \frac{1}{|x+yz|\sqrt{(x+yz)^2-1}}} \quad \boxed{\frac{\partial f}{\partial y} = \frac{z}{|x+yz|\sqrt{(x+yz)^2-1}}} \quad \boxed{\frac{\partial f}{\partial z} = \frac{y}{|x+yz|\sqrt{(x+yz)^2-1}}}$$

(44) $h(x,y) = xe^y + y + 1$

$$\frac{\partial h}{\partial x} = e^y$$

$$\frac{\partial h}{\partial y} = xe^y + 1$$

$$\boxed{\frac{\partial^2 h}{\partial x^2} = 0} \quad \boxed{\frac{\partial^2 h}{\partial y^2} = xe^y} \quad \boxed{\frac{\partial^2 h}{\partial x \partial y} = e^y} \quad \boxed{\frac{\partial^2 h}{\partial y \partial x} = e^y}$$

(52) $w = e^x + x\ln y + y\ln x$

$$w_x = e^x + \ln y + \frac{y}{x} \qquad w_y = \frac{x}{y} + \ln x$$

$$\boxed{w_{xy} = 0 + \frac{1}{y} + \frac{1}{x}} \qquad \boxed{w_{yx} = \frac{1}{y} + \frac{1}{x}}$$

Equal !!