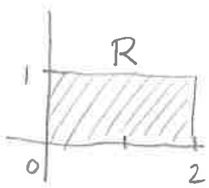
Section 15.1

(2) $\displaystyle\int_0^2\int_{-1}^1 (x-y)\,dy\,dx = \int_0^2 \left[xy - \frac{y^2}{2}\right]_{-1}^1 dx$

$\displaystyle= \int_0^2 \left(x(1) - \frac{1^2}{2}\right) - \left(x(-1) - \frac{(-1)^2}{2}\right) dx$

$\displaystyle= \int_0^2 x - \frac{1}{2} + x + \frac{1}{2}\,dx = \int_0^2 2x\,dx = \left[x^2\right]_0^2 = \boxed{4}$

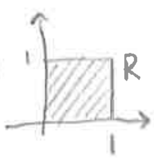
(18) 

$\displaystyle\iint_R xye^{xy^2}\,dA = \int_0^2\int_0^1 xye^{xy^2}\,dy\,dx$

$\displaystyle= \frac{1}{2}\int_0^2\int_0^1 e^{xy^2}2xy\,dy\,dx = \frac{1}{2}\int_0^2 \left[e^{xy^2}\right]_0^1 dx$

$\displaystyle= \frac{1}{2}\int_0^2 (e^x - e^0)\,dx = \frac{1}{2}\int_0^2 (e^x - 1)\,dx$

$\displaystyle= \frac{1}{2}\left[e^x - x\right]_0^2 = \frac{1}{2}\left((e^2-2)-1\right) = \boxed{\frac{e^2-3}{2}}$

(20) 

$\displaystyle\iint_R \frac{y}{x^2y^2+1}\,dA = \int_0^1\int_0^1 \frac{y}{1+(xy)^2}\,dx\,dy$

$\displaystyle= \int_0^1 \left[\tan^{-1}(xy)\right]_0^1 dy = \int_0^1 \tan^{-1}(x)\,dx = \left[x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)\right]_0^1$

$\displaystyle= 1\tan^{-1}(1) - \frac{\ln 2}{2}$

$\displaystyle= \boxed{\frac{\pi}{4} - \frac{\ln 2}{2}}$

$\displaystyle\int \tan^{-1}x\,dx = \int u\,dv = uv - \int v\,du = x\tan^{-1}(x) - \int \frac{x}{1+x^2}\,dx$

$\displaystyle= x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2)$

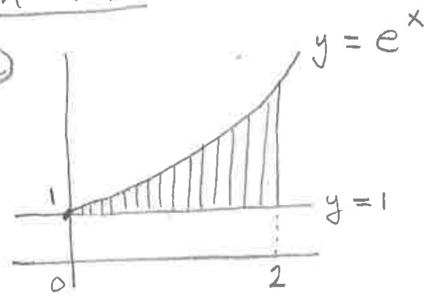
$u = \tan^{-1}x \longrightarrow du = \frac{1}{1+x^2}dx$

$dv = dx \longrightarrow v = x$

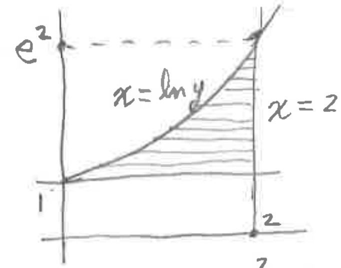
(26) $\displaystyle\int_0^4\int_0^2 \frac{y}{2}\,dy\,dx = \int_0^4 \left[\frac{y^2}{4}\right]_0^2 dx = \int_0^4 dx = \left[x\right]_0^4 = \boxed{4}$

Section 15.2

12 (a)



(b)



$$\iint_R dA = \int_0^2 \int_1^{e^x} dy\, dx$$
$$= \int_0^2 [y]_1^{e^x} dx$$
$$= \int_0^2 (e^x - 1) dx$$
$$= \left[e^x - x\right]_0^2$$
$$= (e^2 - 2) - (e^0 - 0)$$
$$= e^2 - 2 - 1$$
$$= \boxed{e^2 - 3}$$

answers agree

$$\iint_R dA = \int_1^{e^2} \int_{\ln y}^{2} dx\, dy$$
$$= \int_1^{e^2} [x]_{\ln y}^{2} dy$$
$$= \int_1^{e^2} (2 - \ln y)\, dy$$
$$= \left[2y - (y \ln y - y)\right]_1^{e^2}$$
$$= 2e^2 - (e^2 \ln e^2 - e^2) - (2 - (1 \ln 1 - 1))$$
$$= 2e^2 - 2e^2 + e^2 - 2 + 0 - 1$$
$$= \boxed{e^2 - 3}$$

Note Part (b) above required the integral $\int \ln y\, dy$. If you've forgotten what this is you can find it by integration by parts:

$$\int \ln y\, dy = \int u\, dv = uv - \int v\, du$$
$$= (\ln y) y - \int y \frac{1}{y} dy$$
$$= y \ln y - \int dy$$
$$= \boxed{y \ln y - y}$$

$u = \ln y \rightarrow du = \frac{1}{y} dy$

$dv = dy \rightsquigarrow v = y$

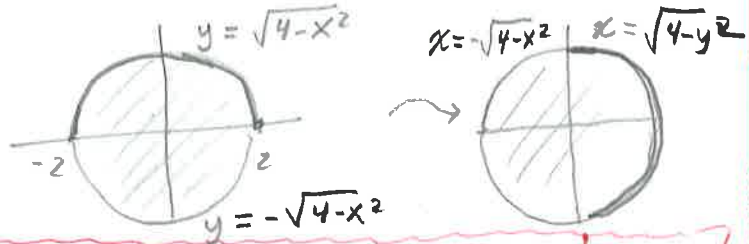
Recall $\sin^2 x = \frac{1 - \cos 2x}{2}$

20

$$\int_0^{\pi} \int_0^{\sin x} y\, dy\, dx = \int_0^{\pi} \left[\frac{y^2}{2}\right]_0^{\sin x} dx = \int_0^{\pi} \frac{1}{2} \sin^2 x\, dx$$
$$= \frac{1}{2} \int_0^{\pi} \frac{1 - \cos 2x}{2} dx = \frac{1}{4} \int_0^{\pi} (1 - \cos 2x)\, dx$$
$$= \frac{1}{4} \left[x - \frac{1}{2} \sin(2x)\right]_0^{\pi} = \frac{1}{4}\left(\pi - \frac{1}{2} \sin 2\pi\right) = \boxed{\frac{\pi}{4}}$$

15.2

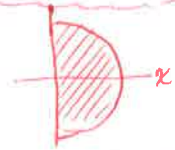
(42) $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x\,dy\,dx$



$\displaystyle = \int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 6x\,dx\,dy$

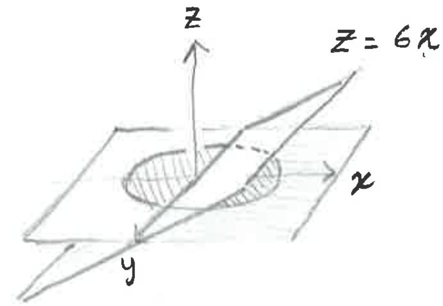
Oops! Misread Problem.

It is $\displaystyle\int_0^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x\,dy\,dx$ → region



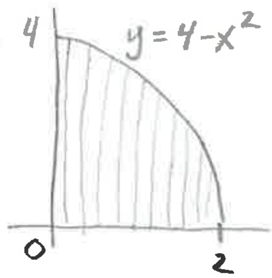
Answer: $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{2-y^2}} 6x\,dx\,dy$ — Sorry! R.H.

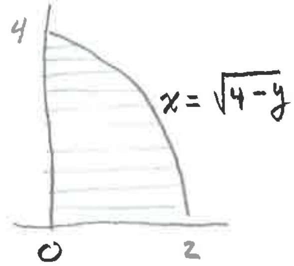
$\displaystyle = \int_{-2}^{2}\left[3x^2\right]_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dy = \int_{-2}^{2} 3(4-y^2)-3(4-y^2)\,dy = \int_0^2 0\,dy = \boxed{0}$



Note The answer of 0 makes sense because the function $z=6$ is positive on the right half of the circle and negative on the left.

(56)





$\displaystyle\int_0^2\int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx$

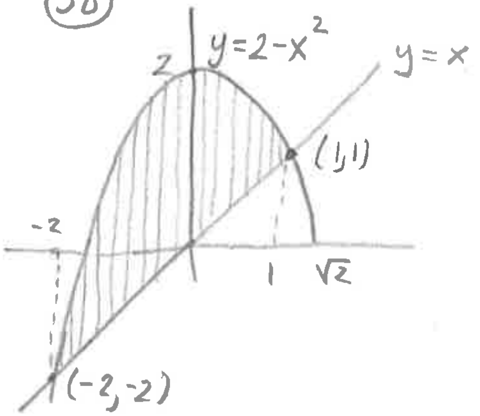
$\displaystyle = \int_0^4\int_0^{\sqrt{4-y}} x\frac{e^{2y}}{4-y}\,dx\,dy = \int_0^4\left[\frac{x^2}{2}\frac{e^{2y}}{4-y}\right]_0^{\sqrt{4-y}} dy$

$\displaystyle = \int_0^4 \frac{4-y}{2}\,\frac{e^{2y}}{4-y}\,dy = \int_0^4 \frac{e^{2y}}{2}\,dy = \left[\frac{e^{2y}}{4}\right]_0^4$

$\displaystyle = \frac{e^8}{4}-\frac{e^0}{4} = \boxed{\frac{e^8-1}{4}}$

Note The problem would be very hard with the original order of integration!

58



$$V = \int_{-2}^{1}\int_{-x}^{2-x^2} x^2\,dy\,dx$$

$$= \int_{-2}^{1}\Big[yx^2\Big]_{x}^{2-x^2}\,dx$$

$$= \int_{-2}^{1}(2-x^2)x^2-(x x^2)\,dx$$

$$= \int_{-2}^{1} 2x^2 - x^4 - x^3\,dx = \left[\frac{2x^3}{3} - \frac{x^5}{5} - \frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left(\frac{2}{3}-\frac{1}{5}-\frac{1}{4}\right) - \left(\frac{2(-2)^3}{3} - \frac{(-2)^5}{5} - \frac{(-2)^4}{4}\right)$$

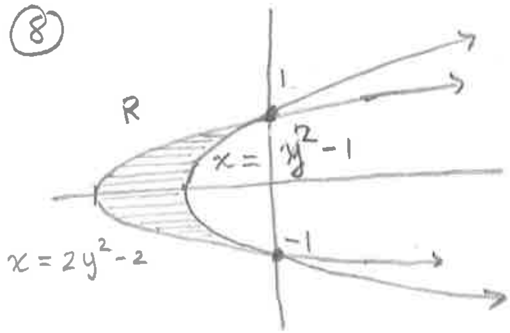
$$= \frac{2}{3} - \frac{1}{5} - \frac{1}{4} + \frac{16}{3} - \frac{32}{5} + \frac{16}{4}$$

$$= \frac{40}{60} - \frac{12}{60} - \frac{15}{60} + \frac{320}{60} - \frac{384}{60} + \frac{240}{60} = \boxed{\frac{189}{60}\text{ cubic units}}$$

$$= \boxed{\frac{63}{20}\text{ cubic units}}$$

Section 15.3

(8)



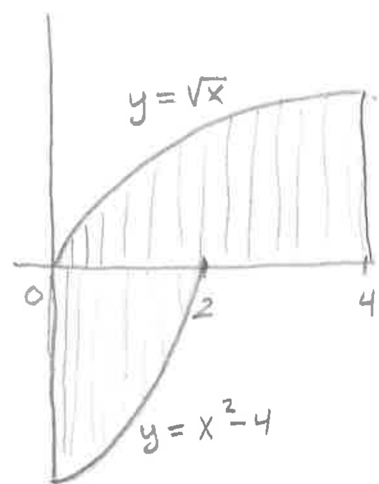
$$\iint_R dA = \int_{-1}^{1} \int_{2y^2-2}^{y^2-1} dx\, dy$$

$$= \int_{-1}^{1} \left[x \right]_{2y^2-2}^{y^2-1} dy = \int_{-1}^{1} y^2 - 1 - 2y^2 + 2 \, dy$$

$$= \int_{-1}^{1} (1 - y^2)\, dy = \left[y - \frac{y^3}{3} \right]_{-1}^{1}$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 - \frac{(-1)^3}{3}\right) = 1 - \frac{1}{3} + 1 - \frac{1}{3} = 2 - \frac{2}{3} = \boxed{\frac{4}{3} \text{ sq. units}}$$

(18) $\displaystyle\int_0^2 \int_{x^2-4}^{0} dy\, dx + \int_0^4 \int_0^{\sqrt{x}} dy\, dx$



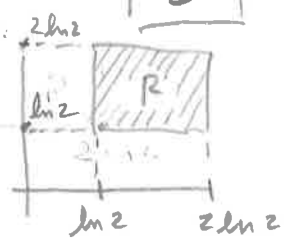
$$= \int_0^2 \left[y \right]_{x^2-4}^{0} dx + \int_0^4 \left[y \right]_0^{\sqrt{x}} dx$$

$$= \int_0^2 (4 - x^2)\, dx + \int_0^4 \sqrt{x}\, dx$$

$$= \left[4x - \frac{x^3}{3} \right]_0^2 + \left[\frac{2\sqrt{x}^3}{3} \right]_0^4$$

$$= 4.2 - \frac{8}{3} + \frac{2\sqrt{4}^3}{3} = 8 - \frac{8}{3} + \frac{16}{3} = 8 + \frac{8}{3} = \frac{24}{3} + \frac{8}{3} = \boxed{\frac{32}{3}}$$

(22) Find average value of $f(x,y) = \dfrac{1}{xy}$ over



Area of square is $(\ln 2)^2$

$$\text{Ave value} = \frac{\iint_R \frac{1}{xy}\, dA}{\text{area of } R} = \frac{\int_{\ln 2}^{2\ln 2} \int_{\ln 2}^{2\ln 2} \frac{1}{xy}\, dy\, dx}{(\ln 2)^2}$$

$$= \frac{1}{(\ln 2)^2} \int_{\ln 2}^{2\ln 2} \left[\frac{1}{x} \ln y \right]_{\ln 2}^{2\ln 2} dx = \frac{1}{(\ln 2)^2} \int_{\ln 2}^{2\ln 2} \frac{1}{x}\left(\ln(2\ln 2) - \ln(\ln 2)\right) dx$$

$$= \frac{1}{(\ln 2)^2} \int_{\ln 2}^{2\ln 2} \frac{1}{x} \ln\left(\frac{2\ln 2}{\ln 2}\right) dx = \frac{\ln 2}{(\ln 2)^2} \int_{\ln 2}^{2\ln 2} \frac{1}{x}\, dx = \frac{\ln 2}{(\ln 2)^2} \left[\ln x \right]_{\ln 2}^{2\ln 2} = \boxed{1}$$