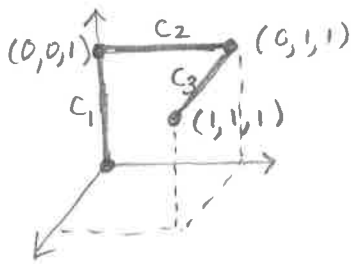
MATH 307 Section 16.1

(16)



$C = C_1 \cup C_2 \cup C_3$

$C_1 = \langle 0, 0, t \rangle \quad 0 \le t \le 1 \rightsquigarrow v(t) = \langle 0,0,1 \rangle \rightsquigarrow |v(t)| = 1$

$C_2 = \langle 0, t, 1 \rangle \quad 0 \le t \le 1 \rightsquigarrow v(t) = \langle 0,1,0 \rangle \rightsquigarrow |v(t)| = 1$

$C_3 = \langle t, 1, 1 \rangle \quad 0 \le t \le 1 \rightsquigarrow v(t) = \langle 1,0,0 \rangle \rightsquigarrow |v(t)| = 1$

$\Big\}\ ds = |v(t)|\,dt = dt$

$$\int_C x + \sqrt{y} - z^2 \, ds$$

$$= \int_{C_1} x + \sqrt{y} - z^2 \, ds + \int_{C_2} x + \sqrt{y} - z^2 \, ds + \int_{C_3} x + \sqrt{y} - z^2 \, ds$$

$$= \int_0^1 0 + \sqrt{0} - t^2 \, dt + \int_0^1 0 + \sqrt{t} - 1^2 \, dt + \int_0^1 t + \sqrt{1} - 1^2 \, dt$$

$$= -\int_0^1 t^2 \, dt + \int_0^1 \sqrt{t} - 1 \, dt + \int_0^1 t \, dt$$

$$= -\left[\frac{t^3}{3}\right]_0^1 + \left[\frac{2}{3}\sqrt{t}^3 - t\right]_0^1 + \left[\frac{t^2}{2}\right]_0^1$$

$$= -\frac{1}{3} + \frac{2}{3} - 1 + \frac{1}{2} = -\frac{2}{6} + \frac{4}{6} - \frac{6}{6} + \frac{3}{6} = \boxed{-\frac{1}{6}}$$

(10) $C: \langle t, 1-t, 1 \rangle, \quad 0 \le t \le 1 \rightsquigarrow v(t) = \langle 1, -1, 0 \rangle$

$\rightsquigarrow |v(t)| = \sqrt{2}, \quad ds = \sqrt{2}\, dt$

$$\int_C x - y + z - 2 \, ds = \int_0^1 (t - (1-t) + 1 - 2)\sqrt{2}\, dt = \int_0^1 (2t - 2)\sqrt{2}\, dt$$

$$= \left[\sqrt{2}t^2 - 2\sqrt{2}t\right]_0^1 = \boxed{-\sqrt{2}}$$

16.1

24 $r(t) = \langle t^3, t^4 \rangle, \quad \frac{1}{2} \le t \le 1$

$V(t) = \langle 3t^2, 4t^3 \rangle$

$|v(t)| = \sqrt{(3t^2)^2 + (4t^3)^2} = \sqrt{9t^4 + 16t^6} = \sqrt{t^4(9+16t^2)} = t^2\sqrt{9+16t^2}$

$$\int_C \frac{\sqrt{y}}{x}\, ds = \int_{\frac{1}{2}}^{1} \frac{\sqrt{t^4}}{t^3}\, t^2 \sqrt{9+16t^2}\, dt = \int_{\frac{1}{2}}^{1} \frac{t^2 t^2}{t^3}\sqrt{9+16t^2}\, dt$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{9+16t^2}\; t\, dt$$

$u = 9 - 16t^2$

$\frac{du}{dt} = -32t$

$du = -32t\, dt$

$$= \frac{1}{32}\int_{\frac{1}{2}}^{1} \sqrt{9+16t^2}\; 32t\, dt$$

$$= \frac{1}{32}\int_{9+16\left(\frac{1}{2}\right)^2}^{9+16\cdot 1^2} \sqrt{u}\, du = \frac{1}{32}\int_{13}^{25} \sqrt{u}\, du$$

$$= \frac{1}{32}\left[\frac{2}{3}\sqrt{u}^3\right]_{13}^{25} = \frac{2}{32\cdot 3}\left[\sqrt{u}^3\right]_{13}^{25}$$

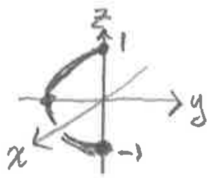
$$= \frac{1}{48}\left(\sqrt{25}^3 - \sqrt{13}^3\right) = \frac{5\sqrt{25} - 13\sqrt{13}}{48}$$

$$= \frac{125 - 13\sqrt{13}}{48}$$

16.1 34 Wire is on curve $\langle 0, t^2-1, 2t \rangle$, $-1 \le t \le 1$

Density at (x,y,z) is $\delta(x,y,z) = 15\sqrt{y+2}$

Find its center of mass.



$\vec{v}(t) = \langle 0, 2t, 2 \rangle$, $|v(t)| = \sqrt{(2t)^2 + 2^2} = 2\sqrt{t^2+1}$

Density at point $\vec{r}(t) = \langle 0, \underbrace{t^2-1}_{y}, 2t \rangle$ on wire

is $\delta(0, t^2-1, 2t) = 15\sqrt{t^2-1+2} = 15\sqrt{t^2+1}$

- MASS $M = \int_C \delta(x,y,z)\,ds = \int_{-1}^{1} 15\sqrt{t^2+1}\,|v(t)|\,dt$

$$= \int_{-1}^{1} 15\sqrt{t^2+1} \cdot 2\sqrt{t^2+1}\,dt = \int_{-1}^{1} 30t^2 + 30\,dt$$

$$= \Big[10t^3 + 30t\Big]_{-1}^{1} = \boxed{80}$$

- Since wire is on yz plane, $\boxed{M_{yz} = 0.}$
- $M_{xz} = \int_C y\,\delta(x,y,z)\,ds = \int_{-1}^{1} (t^2-1) 15\sqrt{t^2-1}\,|v(t)|\,dt$

$$= \int_{-1}^{1} (t^2-1) 15\sqrt{t^2+1}\, 2\sqrt{t^2+1}\,dt = \int_{-1}^{1} 30(t^2-1)(t^2+1)\,dt$$

$$= 30\int_{-1}^{1} (t^4 - 1)\,dt = 30\Big[\frac{t^5}{5} - t\Big]_{-1}^{1}$$

$$= 30\left(\frac{2}{5} - 2\right) = 12 - 60 = -48 \qquad \boxed{M_{xz} = -48}$$

- $M_{xy} = \int_C z\,\delta(x,y,z)\,ds = \int_{-1}^{1} 2t \cdot 15\sqrt{t^2-1+2}\; 2\sqrt{t^2+1}\,dt$

$$= 60\int_{-1}^{1} t(t^2+1)\,dt = 60\int_{-1}^{1} t^3 + t\,dt = 60\Big[\frac{t^4}{4} + \frac{t^2}{2}\Big]_{-1}^{1} = 0$$

- Center of mass: $\left(\frac{M_{yz}}{M}, \frac{M_{xz}}{M}, \frac{M_{xy}}{M}\right) = \left(0, \frac{-48}{80}, 0\right) = \boxed{\left(0, \frac{-3}{5}, 0\right)}$