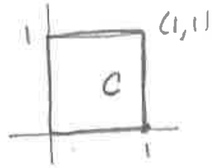
Section 16.4

(6) $\vec{F} = \langle x^2 + 4y,\ x + y^2 \rangle$

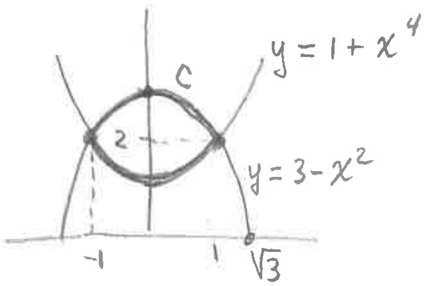


$$\frac{\partial M}{\partial x} = 2x \qquad \frac{\partial N}{\partial x} = 1$$
$$\frac{\partial M}{\partial y} = 4 \qquad \frac{\partial N}{\partial y} = 2y$$

$$\text{Flux} = \oint_C (x^2+4y)\,dy - (x+y^2)\,dx = \iint_R (2x + 2y)\,dA = \int_0^1\int_0^1 (2x+2y)\,dy\,dx$$
$$= \int_0^1 \left[2xy + y^2\right]_0^1 dx = \int_0^1 (2x+1)\,dx = \left[x^2 + x\right]_0^1 = \boxed{2}$$

$$\text{Circ} = \oint_C (x^2+4y)\,dx + (x+y^2)\,dy = \iint_R (1-4)\,dA = \int_0^1\int_0^1 (-3)\,dy\,dx$$
$$= \int_0^1 \left[-3y\right]_0^1 dx = \int_0^1 -3\,dx = \boxed{-3}$$

(18) $F = \left\langle y + e^x \ln y,\ \frac{e^x}{y} \right\rangle$

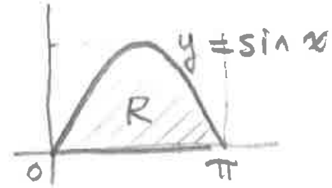


$$\frac{\partial M}{\partial x} = e^x \ln y \qquad \frac{\partial N}{\partial x} = \frac{e^x}{y}$$
$$\frac{\partial M}{\partial y} = 1 + \frac{e^x}{y} \qquad \frac{\partial N}{\partial y} = -\frac{e^x}{y^2}$$

$$\text{Circ} = \oint_C (y + e^x \ln y)\,dx + \frac{e^x}{y}\,dy = \iint_R \left(\frac{e^x}{y} - \left(1 + \frac{e^x}{y}\right)\right) dA$$
$$= \int_{-1}^1 \int_{1+x^4}^{3-x^2} -1\,dy\,dx = \int_{-1}^1 \left[-y\right]_{1+x^4}^{3-x^2} dx$$
$$= \int_{-1}^1 \left(-(3-x^2) + (1+x^4)\right) dx$$
$$= \int_{-1}^1 (x^2 + x^4 - 2)\,dx = \left[\frac{x^3}{3} + \frac{x^5}{5} - 2x\right]_{-1}^1$$
$$= \left(\frac{1}{3} + \frac{1}{5} - 2\right) - \left(-\frac{1}{3} - \frac{1}{5} + 2\right)$$
$$= \frac{2}{3} + \frac{2}{5} - 4 = \frac{10}{15} + \frac{6}{15} - \frac{60}{15} = \boxed{-\frac{44}{15}}$$

(22) $\oint_C 3y\,dx + 2x\,dy$

$M(x,y) = 3y$
$N(x,y) = 2x$

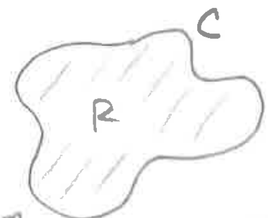


$$= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R (2-3)\,dA = \iint_R (-1)\,dA$$

$$= \int_0^{\pi}\int_0^{\sin x} (-1)\,dy\,dx = \int_0^{\pi} \big[-y\big]_0^{\sin x} dx = \int_0^{\pi} -\sin x\,dx$$

$$= \big[\cos x\big]_0^{\pi} = \cos \pi - \cos 0 = -1-1 = \boxed{-2}$$

(24) $\oint_C (2x+y^2)\,dx + (2xy+3y)\,dy$



any closed curve

$$= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$M(x,y) = 2x+y^2$
$N(x,y) = 2xy+3y$

$$= \iint_R (2y - 2y)\,dA$$

$$= \iint_R 0\,dA = \boxed{0}$$