

VCU

MATH 307

MULTIVARIATE CALCULUS

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TEST 3



April 2, 2014

Name: _____

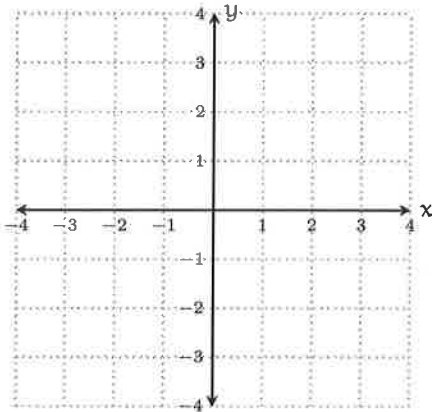
Score: _____

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (20 pts.) This question concerns the integral $\int_0^\pi \int_x^\pi \frac{\sin(y)}{y} dy dx$.

(a) Sketch the region of integration on the coordinate axis below.



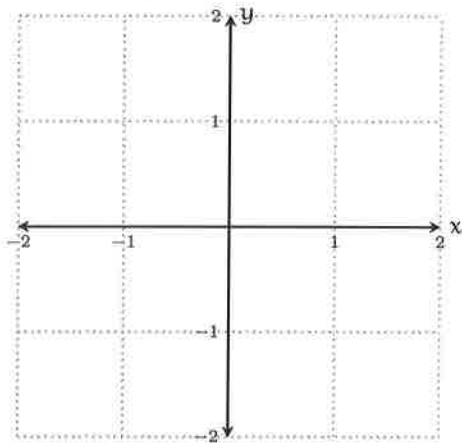
(b) Rewrite the integral with its order of integration reversed.

(c) Evaluate the integral from part (b) above.

2. (20 pts.) Evaluate $\iint_R \frac{8y}{x} dA$, where R is the region on the xy -plane between the graphs of $y = x^2$ and $y = 3x$.

3. (20 pts.) This question concerns the integral $\int_0^1 \int_0^x \frac{y}{x\sqrt{x^2 + y^2}} dy dx$.

(a) Sketch the region of integration on the coordinate axis below.



(b) Rewrite the integral so that it is in polar form.

(c) Evaluate the integral.

4. (20 pts.) Let D be the sphere of radius 2 centered at the origin.

We know from geometry that its volume is $V = \frac{4}{3}\pi 2^3 = \frac{32\pi}{3}$.

Compute this volume by evaluating the integral $V = \iiint_D dV$.

You may find spherical coordinates most convenient.

5. (20 pts.) Suppose D is the cube in the first octant, bounded by the three coordinate planes, and the planes $x = 2$, $y = 2$, and $z = 2$. Find the average value of $f(x, y, z) = 3x^2$ on D .