VCU

MATH 307

Multivariate Calculus

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Sample Test 2

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Name: _____

Score: _____

Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a box, where appropriate.

6. (10 pts.) Find the equation of the tangent plane to $f(x,y) = 2x^4 - xy^2 + 3y^2$ at the point $(1,1,4)$.

Good Luck!

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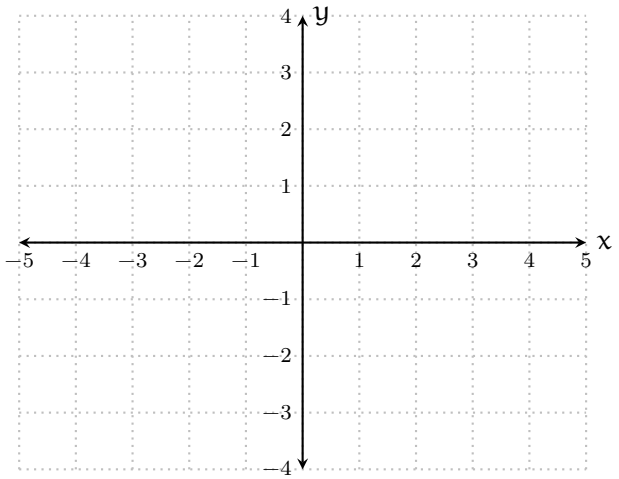
1. (25 points) Consider the function $z = f(x,y) = xy - x$.

(a) What is the domain of f?

(b) Sketch the level curves for $z = 1$ and $z = 0$.



(c) $\nabla f(x,y) =$

(d) Find the rate of change of $f(x,y)$ in the direction of $\langle 3,5\rangle$ at the point $(7,3)$.

2. (20 pts.) Evaluate each limit, if possible; if not, explain why it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{5x^3 - 5y^2x}{x^2 - yx}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2 - 2y^2}$

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3. (15 pts.) Find the maximum and minimum values (and their locations) of the function $f(x, y) = x^2 + y^2$ subject to the constraint $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$.

4. (15 pts.) Suppose $f(x, y)$ is a function for which $\nabla f(15, 2) = \langle 6, -3 \rangle$. Suppose $g(t) = f(t^2 - 1, \sqrt{t})$. Find $g'(4)$.

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5. (15 pts.) Consider $f(x, y) = \dfrac{x^3}{3} - x + y^2$.

Find all critical points; classify them as local maxima, local minima or saddle points.