

4. (25 pts.) Suppose  $D$  is the cylinder whose base is the unit circle on the  $xy$ -plane, and whose top lies on the plane  $z = 2$ .

Compute the integral  $\iiint_D r^2 z^3 \, dV$ .

(Use cylindrical coordinates.)

VCU

MATH 307  
MULTIVARIATE CALCULUS

R. Hammack

TEST 3



November 8, 2013

Name: \_\_\_\_\_

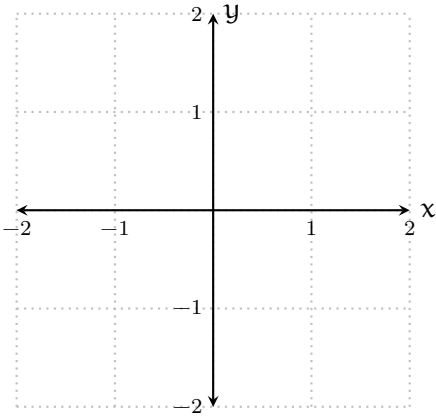
Score: \_\_\_\_\_

**Directions.** Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a  box, where appropriate.

1. (25 points) Consider the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x + 2y) \, dy \, dx.$$

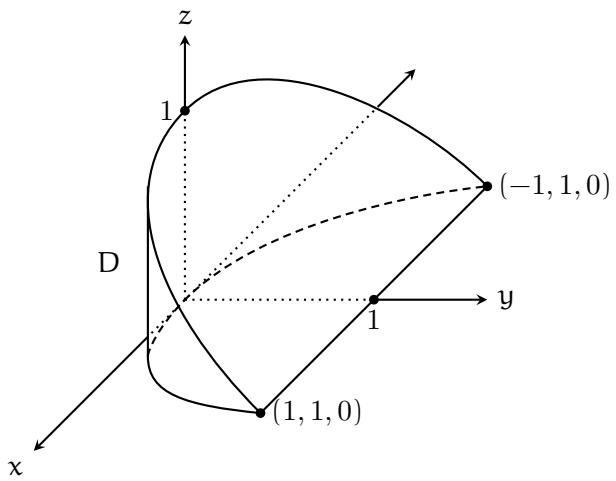
(a) Sketch the region of integration.



(b) Convert the integral to a polar integral.

(c) Evaluate your answer from part (b).

2. (25 pts.) Consider the region  $D$  bounded by the  $xy$ -plane, the graph of  $y = x^2$ , and the plane  $y + z = 1$ .



(a) Set up a triple integral for the volume of  $D$ .

(b) Evaluate the integral to get the volume.

3. (25 pts.) Find the average value of the function  $f(x, y) = \sin(x + y)$  on the rectangle  $0 \leq x \leq \pi$ ,  $0 \leq y \leq \frac{\pi}{2}$ .