

Name: _____

R. Hammack

Score: 12/12**Directions:** Put away phones, calculators, smart watches, etc. Answer in the space provided. Show work.

1. Suppose A, B and C are invertible matrices. Solve the equation $AXC = CB$ for X .

$$\begin{aligned}
 AXC &= CB \\
 A^{-1}AXC &= A^{-1}CB \\
 IXC &= A^{-1}CB \\
 XC &= A^{-1}CB \\
 XCC^{-1} &= A^{-1}CBC^{-1} \\
 XI &= A^{-1}CBC^{-1} \\
 X &= A^{-1}CBC^{-1}
 \end{aligned}$$

2. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 & 5 \\ 1 & 2 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

$$\begin{aligned}
 &\left[\begin{array}{ccc|ccc} 3 & 5 & 5 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \\
 R_1 \leftrightarrow R_2 &\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 1 & 0 \\ 3 & 5 & 5 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \\
 R_2 - 3R_1 \rightarrow R_2 &\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -3 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \\
 \begin{array}{l} R_1 + 2R_2 \rightarrow R_1 \\ R_3 + R_2 \rightarrow R_3 \end{array} &\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & 0 \\ 0 & -1 & -1 & 1 & -3 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array} \right] \\
 R_2 + R_3 \rightarrow R_2 &\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & 0 \\ 0 & -1 & 0 & 2 & -6 & 1 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array} \right] \\
 -R_2 \rightarrow R_2 &\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & 0 \\ 0 & 1 & 0 & -2 & 6 & -1 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array} \right]
 \end{aligned}$$

Thus $A^{-1} = \boxed{\begin{bmatrix} 2 & -5 & 0 \\ -2 & 6 & -1 \\ 1 & -3 & 1 \end{bmatrix}}.$