MATH 501: Abstract Algebra | Test#2 | November 18, 2010

Name: ______________________ R. Hammack Score: ________

Directions: This is a closed-book, closed-notes test. Please answer in the space provided. Explain your reasoning. Use calculators, computers, etc, is not permitted on this test.

1. Find the order of $(8, 8, 8)$ in $\mathbb{Z}_{10} \times \mathbb{Z}_{24} \times \mathbb{Z}_{80}$.

2. Prove or disprove: $\mathbb{Z} \cong \mathbb{Q}$.

3. Suppose H is a normal subgroup of a group G. Prove or disprove:
   If H and G/H are both abelian, then G is also abelian.

4. List all abelian groups of order 360, up to isomorphism.

5. Suppose R and S are rings with multiplicative identities $1_R \in R$ and $1_S \in S$. Prove that if $\varphi : R \to S$ is a surjective ring homomorphism, then $\varphi(1_R) = 1_S$.

6. Suppose G and H are groups. Prove that $G \times H \cong H \times G$.

7. Describe all the homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}_6$.

8. Prove that if $\varphi : G \to H$ is a group homomorphism and G is cyclic, then the subgroup $\varphi(G)$ is cyclic.

9. If a and b are elements in a ring R, then $a(-b) = -(ab)$.

10. Suppose R is an integral domain whose only ideals are $\{0\}$ and R. Prove that R must be a field.