Abstract Algebra	FINAL EXAM	May 23, 2003
Name: ____________________	R. Hammack	Score: ______

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is **not** allowed on this test.

1. Draw the subgroup lattice for $\mathbb{Z}_{18}$.

2. List the elements of the cyclic subgroup $\langle$-i$\rangle$ of $\mathbb{C}^*$.

3. Find the order of the largest cyclic subgroup of the symmetric group S_{10}.
   Consider the element (1,2,3,4,5)(6,7,8)(9,10).

4. Consider the set $H = \{\sigma \in S_5 \mid \sigma(3)=3\}$.

   (a) $|H| =$

   (b) Explain why H is a subgroup of S_5.

   (c) Is H a normal subgroup of S_5? Explain.

   (d) How many left cosets of H are there in S_5?

5. List all the nonisomorphic groups of order 180.

6. Find the order of (3,6,9) in $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$.

7. Are the groups $\mathbb{Z}_8\times\mathbb{Z}_{10}\times\mathbb{Z}_3$ and $\mathbb{Z}_8\times\mathbb{Z}_2\times\mathbb{Z}_{15}$ isomorphic? Why or why not?

8. Find the kernel of the homomorphism $\phi:\mathbb{Z}\to\mathbb{Z}_8$ for which $\phi(1)=6$.

9. Find the kernel of the homomorphism $\phi:\mathbb{Z}_{40}\to\mathbb{Z}_5\times\mathbb{Z}_8$ for which $\phi(1)=(1,4)$.

10. (a) List the units in the ring $\mathbb{Z}_{12}$.

    (b) List the zero divisors in the ring $\mathbb{Z}_{12}$.

    (c) List the prime ideals in the ring $\mathbb{Z}_{12}$.

11. What familiar group is $(\mathbb{Z}_4 \times \mathbb{Z}_6)/\langle\ (2,3)\ \rangle$ isomorphic to?

12. Explain why $\mathbb{C}^*/U \cong \mathbb{R}^+$.

13. Is $2x^3 + x^2 + 2x + 2$ an irreducible polynomial in $\mathbb{Z}_5[x]$? If not, write it as a product of irreducible polynomials.

14. Find all $c \in \mathbb{Z}_3$ for which $\mathbb{Z}_3$[x]/$\langle$ x^2+c $\rangle$ is a field.

15. Prove that if G is a finite group with identity e, and $m = |G|$, then $x^m = e$ for any element $x \in G$.

16. Suppose that G is a group with identity e. Prove that if $x^2 = e$ for every element x in G, then G is abelian.

17. Prove that if G is an abelian group, then the set of all elements $x \in G$ for which $x^2 = e$ form a subgroup of G.

18. Prove that the units of a ring with unity form a multiplicative group.