Name: Richard

QUIZ 11

MATH 300
September 26, 2024

1. Suppose $a, b \in \mathbb{Z}$. Prove the following statement with contrapositive proof. Use completely formed sentences. Use definitions when appropriate.

Proposition: If $25 \nmid ab$, then $5 \nmid a$ or $5 \nmid b$.

Proof (Contrapositive.) Suppose it is not true that $5 \nmid a$ or $5 \nmid b$. Then (by DeMorgan's law) $5 \mid a$ and $5 \mid b$. Consequently $a = 5k$ and $b = 5\ell$ for some $k, \ell \in \mathbb{Z}$. Thus $ab = 5k \cdot 5\ell = 25k\ell$. That is, $ab = 25m$ for $m = k\ell \in \mathbb{Z}$. Therefore $25 \mid ab$. ■

Name: Richard — Quiz 11 — MATH 300, September 24, 2024

1. Suppose $a, b, c \in \mathbb{Z}$. Prove the following statement with contrapositive proof. Use completely formed sentences. Use definitions when appropriate.

Proposition: If $a \nmid bc$, then $a \nmid b$ and $a \nmid c$.

Proof: (Contrapositive.) Suppose that it is not true that $a \nmid b$ and $a \nmid c$. Then (by DeMorgan's Law) $a | b$ or $a | c$. Let's now consider cases.

CASE I Suppose $a|b$. Then $b = ak$ for some $k \in \mathbb{Z}$, and so $bc = akc = a(kc)$. But this means $a | bc$.

CASE II Suppose $a|c$. Then $c = ak$ for some $k \in \mathbb{Z}$, and so $bc = bak = a(bk)$. Again, this means $a|bc$.

In either case, we got $a|bc$.

Therefore $a|bc$