Name: Richard

QUIZ 12

MATH 300
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1. Suppose $a \in \mathbb{Z}$. Prove the following statement.
Use completely formed sentences. Use definitions when appropriate.

Proposition: $a^3 + a^2 + a$ is even if and only if a is even.

Proof First we will prove that if a^3+a^2+a is even, then a is even. For this we will use contrapositive. Suppose that a is not even, That is a is odd. Then $a = 2k+1$ for some $k \in \mathbb{Z}$, and Then

$$
\begin{aligned}
a^3+a^2+a &= (2a+1)^3 + (2a+1)^2 + (2a+1) \\
&= 8a^3 + 12a^2 + 6a + 1 + 4a^2 + 4a + 1 + 2a + 1 \\
&= 8a^3 + 16a^2 + 10a + 2 + 1 \\
&= 2(4a^3 + 8a^2 + 5a + 1) + 1
\end{aligned}
$$

Thus $a^3+a^2+a = 2c+1$ for $c = 4a^3+8a^2+5a+1$, and therefore a^3+a^2+a is odd and not even.

Conversely we will prove that if a is even then a^3+a^2+a is even. We will use direct proof. Suppose a is even. Then $a = 2c$ for some $c \in \mathbb{Z}$. Consequently $a^3+a^2+a = (2a)^3 + (2a)^2 + 2a = 8a^3 + 4a + 2a = 2(4a^3+2a+a)$. Thus $a^3+a^2+a = 2c$ for $c = 4a^3+2a+a \in \mathbb{Z}$. Therefore a^3+a^2+a is even.

The proof is complete. $\blacksquare$