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QUIZ 13

MATH 300
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1. Suppose A, B and C are sets.
Prove: $(A \cap B) - C = (A - C) \cap (B - C)$.

Proof First we will show $(A \cap B) - C \subseteq (A - C) \cap (B - C)$.
Suppose $x \in (A \cap B) - C$. This means $x \in A \cap B$ and $x \notin C$. But since $x \in A \cap B$, we know $x \in A$ and $x \in B$.
Because $x \in A$ and $x \notin C$, we get $x \in A - C$.
Because $x \in B$ and $x \notin C$, we get $x \in B - C$.
But then, as $x \in A - C$ and $x \in B - C$, it must be that $x \in (A - C) \cap (B - C)$.

This shows $(A \cap B) - C \subseteq (A - C) \cap (B - C)$.

Next we will show $(A - C) \cap (B - C) \subseteq (A \cap B) - C$.
Suppose $x \in (A - C) \cap (B - C)$. Then, by definition of intersection, $x \in A - C$ and $x \in B - C$. And by the definition of set difference, this gives $x \in A$ and $x \notin C$ and $x \in B$ and $x \notin C$.
Because $x \in A$ and $x \in B$, we get $x \in A \cap B$.
But, in addition, $x \notin C$, so $x \in (A \cap B) - C$.

This shows $(A - C) \cap (B - C) \subseteq (A \cap B) - C$.

Having shown $(A \cap B) - C \subseteq (A - C) \cap (B - C)$ and $(A - C) \cap (B - C) \subseteq (A \cap B) - C$, we have now established $(A \cap B) - C = (A - C) \cap (B - C)$ ∎