Name: Richard QUIZ 14 MATH 300
October 17, 2024

1. Prove or disprove. If A and B are sets, then $\mathscr{P}(A \cap B) \subseteq \mathscr{P}(A) \cap \mathscr{P}(B)$.

This is TRUE

Proof Let $X \in \mathscr{P}(A \cap B)$. This means $X \subseteq A \cap B$.
So every element of X is in A and in B.
Therefore $X \subseteq A$ and $X \subseteq B$.
Hence $X \in \mathscr{P}(A)$ and $X \in \mathscr{P}(B)$
Consequently $X \in \mathscr{P}(A) \cap \mathscr{P}(B)$.
Therefore $\mathscr{P}(A \cap B) \subseteq \mathscr{P}(A) \cap \mathscr{P}(B)$.

2. Prove or disprove. If A and B are sets, then $\mathscr{P}(A \cup B) \subseteq \mathscr{P}(A) \cup \mathscr{P}(B)$.

This is FALSE

For a counterexample, let $A = \{1\}$ and $B = \{2\}$.
Then $\mathscr{P}(A \cup B) = \mathscr{P}(\{1,2\}) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$.
But $\mathscr{P}(A) \cup \mathscr{P}(B) = \{\phi, \{1\}\} \cup \{\phi, \{2\}\} = \{\phi, \{1\}, \{2\}\}$.
This shows $\mathscr{P}(A \cup B) \not\subseteq \mathscr{P}(A) \cup \mathscr{P}(B)$ in general.

1. Prove or disprove. If A and B are sets, then $\mathscr{P}(A \cup B) \subseteq \mathscr{P}(A) \cup \mathscr{P}(B)$.

This is <u>FALSE</u>

[See solution of #2 on Heads Quiz]

2. Prove or disprove. If A and B are sets, then $\mathscr{P}(A \cap B) \subseteq \mathscr{P}(A) \cap \mathscr{P}(B)$.

This is <u>TRUE</u>

[See solution of #1 on Tails Quiz].