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QUIZ 15

MATH 300
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1. Use induction to prove: If $n \in \mathbb{N}$, then $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$.

Proof (Induction)

(1) For the basis case, notice that for $n=1$ the statement is $\frac{1}{(1+1)!} = 1-\frac{1}{(1+1)!}$ and this reduces to $\frac{1}{2} = 1-\frac{1}{2}$, which is true.

(2) Now assume that the statement is true for some $n=k\geq 1$. That is, assume that

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{k}{(k+1)!} = 1-\frac{1}{(k+1)!} \quad (*)$$

We need to prove it for $n=k+1$. Observe:

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{k}{(k+1)!}+\frac{k+1}{(k+1+1)!}$$

$$= 1-\frac{1}{(k+1)!}+\frac{k+1}{(k+2)!}$$

$$= 1-\frac{(k+2)}{(k+2)(k+1)!}+\frac{k+1}{(k+2)!}$$

$$= 1-\frac{k+2}{(k+2)!}+\frac{k+1}{(k+2)!} = 1+\frac{-(k+2)+(k+1)}{(k+2)!}$$

$$= 1-\frac{1}{(k+2)!} = 1-\frac{1}{((k+1)+1)!}$$

This shows $\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{k+1}{(k+1+1)!} = 1-\frac{1}{((k+1)+1)!}$

That is, the statement is true for $n=k+1$