

1. How many different functions
- $f: \mathbb{Z}_5 \rightarrow \mathbb{Z}_3$
- are there?

In constructing an arbitrary function $f: \mathbb{Z}_5 \rightarrow \mathbb{Z}_3$, there are 3 choices for $f([0])$, namely $[0], [1], [2]$
 3 choices for $f([1])$, namely $[0], [1], [2]$
 3 choices for $f([2])$, " " " "
 3 choices for $f([3])$, " " " "
 3 choices for $f([4])$, " " " "

By the multiplication principle there are $3^5 = 243$ functions $\mathbb{Z}_5 \rightarrow \mathbb{Z}_3$

2. Is the set
- $f = \{(x^2, x) : x \in \mathbb{R}\}$
- a function? If so, what is its domain and range? Explain.

No this is not a function because $(4, 2)$ and $(4, -2)$ are both in f , so f contains more than one ordered pair with first coordinate 4.

3. Is the set
- $\theta = \{(X, |X|) : X \subseteq \mathbb{Z}_5\}$
- a function? If so, what is its domain and range? Explain.

Yes $\theta \subseteq \mathcal{P}(\mathbb{Z}_5) \times \{0, 1, 2, 3, 4, 5\}$, and for any $X \in \mathcal{P}(\mathbb{Z}_5)$, θ contains only one ordered pair $(X, |X|)$ whose 1st coordinate is X .

Domain of θ is $\mathcal{P}(\mathbb{Z}_5)$

Range of θ is $\{0, 1, 2, 3, 4, 5\}$