Name: Richard

QUIZ 19

MATH 300
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1. Let $\theta : \mathscr{P}(\mathbb{Z}) \to \mathscr{P}(\mathbb{Z})$ be defined as $\theta(X) = \overline{X}$. Explain your answers to the following questions.

(a) Is θ injective? Yes!

Proof (Contrapositive) Suppose $X, Y \in \mathscr{P}(\mathbb{Z})$ and $\theta(X) = \theta(Y)$. This means $\overline{X} = \overline{Y}$, so $\overline{\overline{X}} = \overline{\overline{Y}}$, which simplifies to $X = Y$.

(b) Is θ surjective? Yes!

Proof Take any B in the codomain $\mathscr{P}(\mathbb{Z})$. (So $B \subseteq \mathbb{Z}$). Then $\overline{B}$ belongs to the domain $\mathscr{P}(\mathbb{Z})$ and $\theta(\overline{B}) = \overline{\overline{B}} = B$.

(c) Is θ bijective?

Yes because its both injective and surjective.

2. This question concerns functions $f : \{A, B, C, D, E, F, G\} \to \{1, 2, 3, 4, 5, 6, 7\}$.

(a) How many such functions are there?

In making such a function, there are 7 choices for $f(A)$, 7 for $f(B)$, etc.

x	A	B	C	D	E	F	G
$f(x)$	$\boxed{7}$	$\boxed{7}$	$\boxed{7}$	$\boxed{7}$	$\boxed{7}$	$\boxed{7}$	$\boxed{7}$

So there are $7^7 = 823{,}543$ functions all together.

(b) How many of these functions are injective?

The reasoning is the same as above, except that once we've decided $f(A)$, this value cannot be used again, so there are only 6 choices for $f(B)$, etc.

x	A	B	C	D	E	F	G
$f(x)$	$7 \cdot$	$6 \cdot$	$5 \cdot$	$4 \cdot$	$3 \cdot$	$2 \cdot$	1

Thus there are $7!$ injective functions.