
Name: _____

QUIZ 20 

MATH 300
November 21, 2024

Directions. This is a take-home quiz. You may consult the textbook, but do not get help from any other resource or person. Turn in the completed quiz at the beginning of class on Tuesday December 3.

1. Consider the functions $f, g : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ defined as $f(m, n) = (3m - 4n, 2m + n)$ and $g(m, n) = (5m + n, m)$. Find the formulas for $g \circ f$ and $f \circ g$.

2. The function $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ defined by the formula $f(m, n) = (5m + 4n, 4m + 3n)$ is bijective. Find its inverse.

3. Given a function $f : A \rightarrow B$ and a subset $Y \subseteq B$, is $f(f^{-1}(Y)) = Y$ always true? Prove or give a counterexample.

4. Given a function $f : A \rightarrow B$ and subsets $W, X \subseteq A$, prove $f(W \cap X) \subseteq f(W) \cap f(X)$.