Name: Richard

QUIZ 6

MATH 300
September 10, 2024

1. This question concerns the statement $R: \quad \forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^2 = a$

(a) Is this statement true or false?

False (because if a is negative there is no b for which $b^2 = a$.)

(b) Form the negation R, and simplify.

$$\sim R: \quad \sim(\forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^2 = a)$$
$$= \exists a \in \mathbb{R}, \sim(\exists b \in \mathbb{R}, b^2 = a)$$
$$= \exists a \in \mathbb{R}, \forall b \in \mathbb{R} \sim(b^2 = a)$$
$$= \boxed{\exists a \in \mathbb{R}, \forall b \in \mathbb{R}, b^2 \neq a}$$

(c) Write R as an English sentence.

For any real number a, there is a real number b for which $b^2 = a$

(d) Write $\sim R$ as an English sentence.

There exists a real number a for which $b^2 \neq a$ for all real numbers b.

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1. This question concerns the statement $R: \quad \forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^3 = a$

(a) Is this statement true or false?

True, because it asserts that any real number has a cube root, which is true.

(b) Form the negation R, and simplify.

$$\begin{aligned} \sim R &= \sim(\forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^3 = a) \\ &= \exists a \in \mathbb{R}, \sim(\exists b \in \mathbb{R}\ b^3 = a) \\ &= \exists a \in \mathbb{R}, \forall b \in \mathbb{R}\ \sim(b^3 = a) \\ &= \boxed{\exists a \in \mathbb{R}, \forall b \in \mathbb{R}\ b^3 \neq a} \end{aligned}$$

(c) Write R as an English sentence.

For every real number a, there exists a real number b for which $b^3 = a$

(d) Write $\sim R$ as an English sentence.

There is a real number a for which $a \neq b^3$ for all real numbers b.