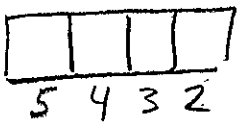
Name: Richard

QUIZ 7

MATH 300
September 12, 2024

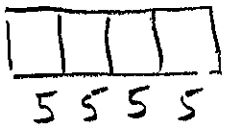
1. This problem concerns lists made from the five digits 1, 2, 3, 4, 5.

(a) How many length-4 lists are there if repetition is **not** allowed?



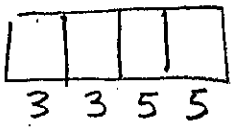
Ans. $5 \cdot 4 \cdot 3 \cdot 2 =$ **120**

(b) How many length-4 lists are there if repetition is allowed?



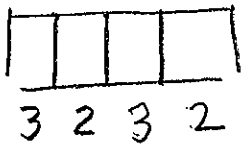
Ans $5 \cdot 5 \cdot 5 \cdot 5 =$ **625**

(c) How many length-4 lists are there if repetition is allowed, and the first two entries are odd?



Ans $3 \cdot 3 \cdot 5 \cdot 5 =$ **225**

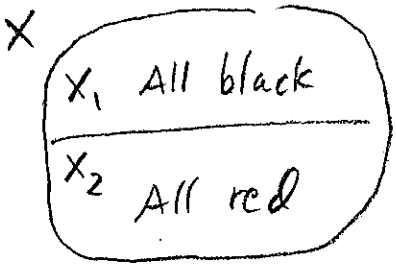
(d) How many length-4 lists are there if repetition is **not** allowed, and the first two entries odd?



Ans $3 \cdot 2 \cdot 3 \cdot 2 =$ **36**

2. Five cards are dealt off of a shuffled 52-card deck and lined up in a row.

(a) How many such 5-card lineups are there in which all five cards have the same color? (i.e., all red, or all black)



Let X be the set of 5-card lineups having all cards the same color. Let X_1 be the all-red lineups and let X_2 be the all-black lineups.

Answer is $|X| = |X_1| + |X_2| =$

$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 + 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$ = 15,787,200

(b) How many such 5-card lineups are there in which **not** all five have the same color?

Let U be the set of all possible 5-card lineups. Let X be the same set as in part (a) above. By the subtraction principle, our answer is

$|\overline{X}| = |U| - |X| = 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 - 2(26 \cdot 25 \cdot 24 \cdot 23 \cdot 22)$

= 296,088,000

Name: Richard

QUIZ 7

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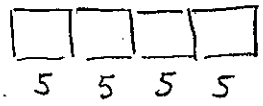
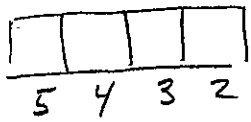
1. This problem concerns lists made from the five symbols C, O, U, N, T.

(a) How many length-4 lists are there if repetition is allowed?



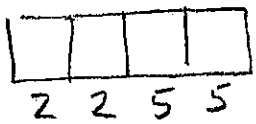
Ans $5^4 = \boxed{625}$

(b) How many length-4 lists are there if repetition is **not** allowed?



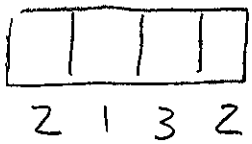
Ans. $5 \cdot 4 \cdot 3 \cdot 2 = \boxed{120}$

(c) How many length-4 lists are there if repetition is allowed, and the first two entries are vowels?



Ans. $2 \cdot 2 \cdot 5 \cdot 5 = \boxed{100}$

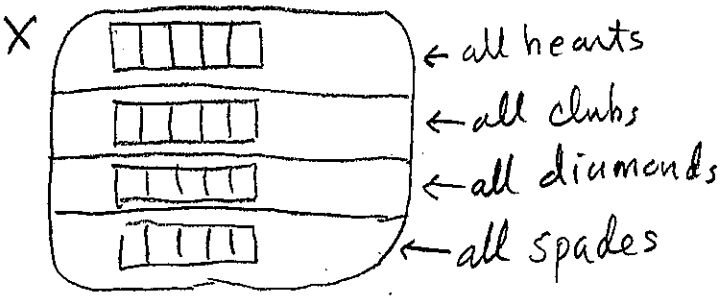
(d) How many length-4 lists are there if repetition is **not** allowed, and the first two entries are vowels?



Ans. $2 \cdot 1 \cdot 3 \cdot 2 = \boxed{12}$

2. Five cards are dealt off of a shuffled 52-card deck and lined up in a row.

(a) How many such 5-card lineups are there in which all five cards are of the same suit?

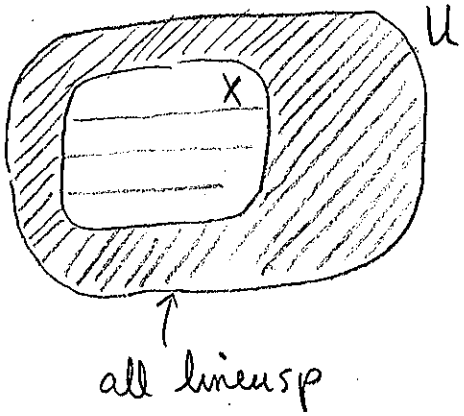


By the addition principle the answer is

$4 \cdot (13 \cdot 12 \cdot 11 \cdot 10 \cdot 9)$

$= 617{,}760$

(b) How many such 5-card lineups are there in which **not** all five cards are of the same suit?



Let U be the set of all 5-card lineups. Let X be the set of lineups in which all 5 cards have the same suit.

By the subtraction principle, our answer is $|U| - |X| =$

$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 - 4(13 \cdot 12 \cdot 11 \cdot 10 \cdot 9) = 311{,}257{,}440$