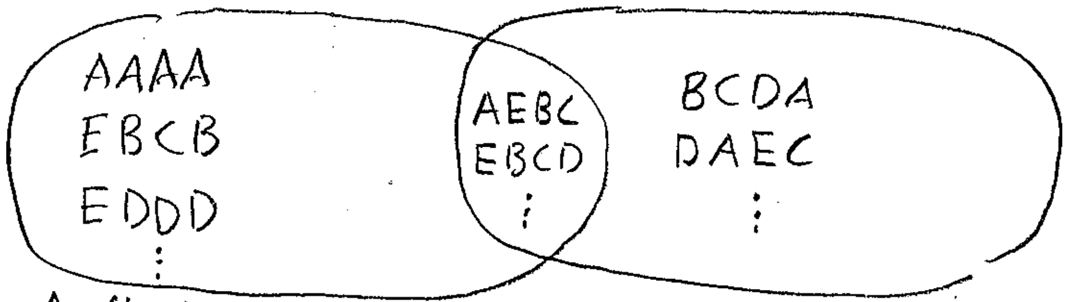
Name: Richard QUIZ 9 MATH 300 September 19, 2024

1. A length-4 list is made from the letters A, B C, D, E, with repetition allowed. How many such lists begin with a vowel **or** have no repeated letters? (Examples: EDCC, EAAA, ABCD, DCAE, BCDE.)



A (begin with vowel) B (no repetition)

Answer: $|A \cup B| = |A| + |B| - |A \cap B|$

$= 2 \cdot 5^3 + 5 \cdot 4 \cdot 3 \cdot 2 - 2 \cdot 4 \cdot 3 \cdot 2$

$= 250 + 120 - 48 = \boxed{322}$

2. Use the binomial theorem to show why $3^n = 2^0\binom{n}{0} + 2^1\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + \cdots + 2^n\binom{n}{n}$

Just note that by the binomial theorem

$$3^n = (1+2)^n = \binom{n}{0}1^n 2^0 + \binom{n}{1}1^{n-1} \cdot 2^1 + \binom{n}{2}1^{n-2} \cdot 2^2 + \cdots + \binom{n}{n}1^0 \cdot 2^n$$

$$= 2^0\binom{n}{0} + 2^1\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}.$$

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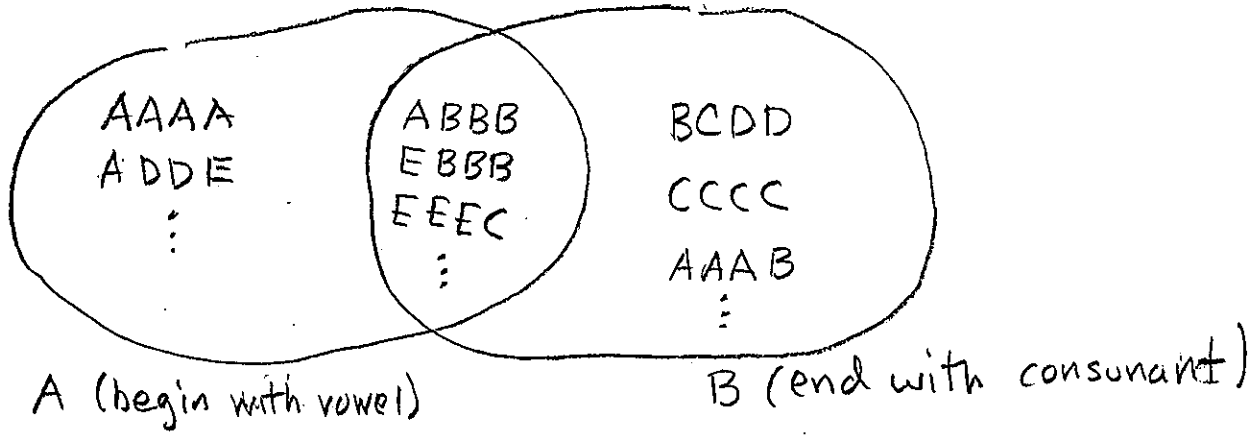
QUIZ 9

MATH 300
September 19, 2024

1. A length-4 list is made from the letters A, B C, D, E, with repetition allowed. How many such lists begin with a vowel or end with a consonant?



$$\underline{\text{Ans:}}\ |A\cup B| = |A| + |B| - |A\cap B|$$
$$= 2\cdot 5^3 + 5^3\cdot 3 - 2\cdot 5^2\cdot 3$$
$$= 250 + 375 - 150 \quad = \boxed{475}$$

2. Use the binomial theorem to show why $4^n = 3^0\binom{n}{0} + 3^1\binom{n}{1} + 3^2\binom{n}{2} + 3^3\binom{n}{3} + \cdots + 3^n\binom{n}{n}$

Just note that by the binomial theorem

$$4^n = (1+3)^n = \binom{n}{0}1^n 3^0 + \binom{n}{1}1^{n-1}3^1 + \binom{n}{2}1^{n-2}3^2 + \cdots + \binom{n}{n}1^0 3^n$$
$$= 3^0\binom{n}{0} + 3^1\binom{n}{1} + 3^2\binom{n}{2} + \cdots 3^n\binom{n}{n}.$$