MATH 300 Test #1 October 1, 2024

Name:_____ R. Hammack Score:_____

Directions: Problems 1–3 on Page 1 are short-answer. For all other problems you must show your work. This test is closed-book and closed-notes. No calculators or other electronic devices.

1. (12 points)

   (a) Let $X = \{\ldots -2, 8, 18, 28, 38, 48, 58, \ldots\}$. Write X in set-builder notation.

   (b) $\big\{5n : n \in \mathbb{Z},\ n^2 \le 16\big\} =$

   (c) $\displaystyle\bigcup_{n\in\mathbb{N}} \big\{x \in \mathbb{R} : |x| > 1/n\big\} =$

2. (12 points) Suppose A and B are sets for which $|A| = m$ and $|B| = n$. Find the cardinalities:

   (a) $|\mathscr{P}(A) - \{A\}| =$

   (b) $|\mathscr{P}(A) \times \mathscr{P}(A \times B)| =$

   (c) $\big|\{X \in \mathscr{P}(B) \ : \ |X| = 5\}\big| =$

3. (4 points)

   (a) Here are the first several rows of Pascal's triangle. Write the next row.

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

   (b) Use part (a) to find the coefficient of x^3y^3 in $(2x-y)^6$. Simplify your answer as much as possible.

4. (12 points) This question concerns the following statement.

   For every real number x, there is a real number y for which $xy > x$.

   (a) Is this statement true or false? Explain.

   (b) Write the statement in symbolic form.

   (c) Form the negation of your answer from (b) above, and simplify.

   (d) Write the negation from (c) above as a well-formed English sentence.

5. (10 points) How many 10-digit integers have fewer than four 0's?

6. (10 points) How many 5-digit positive integers are there that are even or contain no 0's?

7. (10 points) Suppose $x \in \mathbb{Z}$. Prove: If $x^2 - 6x + 5$ is even, then x is odd. [Use contrapositive.]

8. (10 points) Prove that $\sqrt{2}$ is irrational. [Use contradiction.]

9. (10 points) Prove: If a and b are integers, then $(a+b)^3 \equiv a^3 + b^3 \pmod 3$. [Use direct proof]

10. (10 points) Prove: If $n \in \mathbb{Z}$, then $4 \mid n^2$ or $4 \mid (n^2+3)$.