Introduction to Mathematical Reason	MATH 300 Test 2	October 31, 2024
Name:______________	R. Hammack	Score:______

1. Prove that x is odd if and only if x^2 is odd.

2. Prove: There is a set X for which $\mathbb{N} \in X$ and $\mathbb{N} \subseteq X$.

3. Suppose $A = \{6a + 15b : a, b \in \mathbb{Z}\}$ and $B = \{x \in \mathbb{Z} : 3 \mid x\}$. Prove that $A = B$.

4. Suppose R is a symmetric and transitive relation on a set A, and there is an element $a \in A$ for which aRx for every $x \in A$. Prove that R is reflexive.

5. Suppose $n \in \mathbb{N}$. Use induction to prove that $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$.

6. Use induction or smallest counterexample to prove that $24 \mid (5^{2n} - 1)$ for every integer $n \geq 0$.

7. Prove that if X and Y are sets and $\mathscr{P}(X) \subseteq \mathscr{P}(Y)$, then $X \subseteq Y$.

8. Consider the relation on the set $A = \{1, 2, 3, 4, 5\}$ defined as $R = \{(x, y) : x \text{ is even and } y \text{ is odd}\}$.

   (a) Draw a diagram for R.

   (b) Is R transitive? Explain.

9. Prove or disprove: Every transitive and symmetric relation is reflexive.