Introduction to Mathematical Reason	Test 1 (Sample) MATH 300	March 4, 2011

Name: _____ R. Hammack Score: _____

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is not allowed on this test.

1. **Short answer.** Write each of the following sets by listing its elements or describing it with a familiar symbol or symbols.

   (a) $\{n \in \mathbb{Z} : |n| \leq 2\} \;=$

   (b) $\{x \in \mathbb{R} \,:\, \cos(\pi x) = -1\} \;=$

   (c) $\{X \in \mathscr{P}(\mathbb{N}) \,:\, X \cap \{1,2\} = X\} \;=$

   (d) $\bigcap_{n\in\mathbb{N}} \left[1, 2 + \frac{1}{n}\right] \;=$

   (e) $\mathscr{P}(\{1\}) \times \mathscr{P}(\{2\}) \;=$

2. **Short answer.** Write the following sets in set-builder notation.

   (a) $\{2, -7, 12, -17, 22, -27, 32, -37, \ldots\} =$

   (b) $\left\{\frac{1}{1}, \frac{2}{3}, \frac{3}{9}, \frac{4}{27}, \frac{5}{81}, \ldots\right\} =$

3. Write a truth table to decide if $(\sim P) \Rightarrow Q$ and $(P \wedge Q) \Rightarrow P$ are logically equivalent.

4. This problem concerns the following statement.
   $P:$ For every subset X of $\mathbb{N}$, there is an integer m for which $|X| = m$.

   (a) Is the statement P true or false? Explain.

   (b) Form the negation $\sim P$. Write your answer as an English sentence.

5. Suppose that $(R \Rightarrow S) \vee \sim (P \wedge Q)$ is **false**.
   Is there enough information to determine the truth values of P, Q, R and S? If so, what are they? (This is most easily done without a truth table.)

6. How many 10-digit integers contain no 0's and exactly three 6's?

7. Prove: Suppose $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.
(Suggestion: Try direct proof.)

8. Prove: Suppose $a, b \in \mathbb{Z}$. If $a^2(b^2 - 2b)$ is odd, then both a and b are odd.
(Suggestion: Try contrapositive proof.)

9. Prove: For all integers a and b, $a^2 - 4b - 2 \neq 0$.
(Suggestion: Contradiction may be easiest.)

10. Prove: Suppose $a, b \in \mathbb{Z}$. If $25 \nmid ab$, then $5 \nmid a$ or $5 \nmid b$.