

Name: _____

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Score: _____

PART I. (10 points each) Prove the following statements.

1. Prove that an integer a is even if and only if $a^2 + 2a + 9$ is odd.

2. Suppose A, B and C are nonempty sets. Prove that if $A \times B \subseteq B \times C$, then $A \subseteq C$.

3. Use induction to prove that $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$.

4. There exists a set X for which $\mathbb{Z} \in X$, $\mathbb{N} \in \mathcal{P}(X)$ and $\mathbb{R} \in \mathcal{P}(X)$.

5. Use induction to prove that $24 \mid (5^{2n} - 1)$ for every integer $n \geq 0$.

PART II. (10 points each)

Decide if the following statements are true or false. Prove the true statements; disprove the false ones.

6. If A, B and C are sets, then $A \cup (B - C) = (A \cup B) - (A \cup C)$.

7. Suppose a and b are integers. If $a|b$ and $b|a$, then $a = b$.

8. If A, B, C are finite sets and $A \cap B \cap C = \emptyset$, then $|A \cup B \cup C| = |A| + |B| + |C|$.

PART III. (10 points each)

9. Let $A = \{a, b, c, d, e\}$. Consider the relation $R = \{(a, a), (a, b), (b, a), (b, b), (d, c), (d, e), (c, e)\}$ on A .

(a) Draw a diagram for the relation R .

(b) Is the relation R reflexive?

(c) Is the relation R symmetric?

(d) Is the relation R transitive?

10. Let n be a fixed positive integer. As noted in class, congruence modulo n is a relation on the set \mathbb{Z} . In other words, any two integers a and b are related by this relation if $a \equiv b \pmod{n}$. Prove that this relation is transitive.