Introduction to Mathematical Reason	Test 1 MATH 300	March 6, 2012
Name: ______________	R. Hammack	Score: ______

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other electronic devices is not allowed on this test.

1. **Short answer.** Write each of the following sets by listing its elements between braces or describing it with a familiar symbol or symbols.

   (a) $\displaystyle\bigcap_{n\in\mathbb{N}} \left\{ x \in \mathbb{R} \,:\, \frac{-1}{n} \leq x \leq \frac{1}{n} \right\} \;=$

   (b) $\displaystyle\bigcup_{n\in\mathbb{N}} \left\{ x \in \mathbb{R} \,:\, \frac{-1}{n} \leq x \leq \frac{1}{n} \right\} \;=$

   (c) $\big\{ X \subseteq \{a,b,c,d\} \,:\, |\mathscr{P}(X)| = 8 \big\} \;=$

2. **Short answer.** Write the following sets in set-builder notation.

   (a) $\big\{0, 1,\ 4,\ 9,\ 16,\ 25,\ 36, \ldots\big\} =$

   (b) $\left\{ \ldots, \frac{2}{5}, \frac{1}{2}, 0, \frac{1}{4}, \frac{2}{7}, \frac{3}{10}, \frac{4}{13}, \frac{5}{16}, \frac{6}{19}, \ldots \right\} =$

   (c) $\big\{ \{2\},\ \{2,4\},\ \{2,4,6\},\ \{2,4,6,8\},\ \{2,4,6,8,10\},\ \ldots \big\} =$

3. Are $Q \Rightarrow P$ and $(\sim P) \Rightarrow (Q \wedge \sim Q)$ logically equivalent? Support your answer with a truth table.

4. This problem concerns the following statement.
$P:$ Given any $x \in \mathbb{R}$, there exists an element $y \in \mathbb{R}$ for which $xy = 1$.

   (a) Is the statement P true or false? **Explain**.

   (b) Form the negation $\sim P$. Write your answer as an English sentence.
   (The sentence may use mathematical symbols.)

5. This problem concerns 4-card hands dealt off of a standard 52-card deck. How many 4-card hands are there for which all four cards are of the same suit or all four cards are red?

6. Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $a \mid (b^2 + c)$, then $a \mid c$. (Hint: Try direct.)

7. Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a \nmid bc$, then $a \nmid b$ and $a \nmid c$. (Hint: Try contrapositive.)

8. Prove that $\sqrt{6}$ is irrational. (Hint: Try contradiction.)

9. Suppose $x, y \in \mathbb{R}$. Prove that if $xy - x^2 + x^3 \geq x^2y^3 + 4$, then $x \geq 0$ or $y \leq 0$.

10. Suppose $a, b, c \in \mathbb{Z}$, and $n \in \mathbb{N}$. Prove the following:
If $a \equiv b \pmod{n}$ and $a \equiv c \pmod{n}$, then $2a \equiv b + c \pmod{n}$.