Introduction to Mathematical Reason | MATH 300 Test #2 | April 12, 2011

Name:____________________ R. Hammack Score:_______

1. (14 points) Prove that $x \in \{12a + 45b : a, b \in \mathbb{Z}\}$ if and only if $3 \mid x$.

2. (14 points) Suppose A, B, C and D are sets. Prove that $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$.

3. (14 points) Prove that $\{3a + 5b : a, b \in \mathbb{Z}\} = \mathbb{Z}$.

4. (15 points) Recall that Fibonacci Sequence is defined as $F_1 = 1$, $F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$. Use induction to prove that $F_1^2 + F_2^2 + F_3^2 + F_4^2 + \cdots + F_n^2 = F_n F_{n+1}$.

5. (14 points) Use induction to prove that $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \cdots + \dfrac{n}{(n+1)!} = 1 - \dfrac{1}{(n+1)!}$.

6. (14 points) Prove or disprove:
   If A and B are sets, then $\mathscr{P}(A \cup B) = \mathscr{P}(A) \cup \mathscr{P}(B)$.

7. (15 points) Prove or disprove:
   If R and S are two equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.